NASA TECHNICAL NOTE

NASA TN D-5402

C.1

LOAN COPY: RETURN
AFWL (WLIL-2)
KIRTLAND AFB, N ME

TECH LIBRARY KAFB, NM
0132111

NASA TN D-5402

FORTRAN PROGRAM FOR MACHINE COMPUTATION OF GROUP TABLES OF FINITE GROUPS

by Gabriel Allen, David D. Evans, and Paul Swigert

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1969

Doc out

ERRATA

NASA Technical Note D-5402

FORTRAN PROGRAM FOR MACHINE COMPUTATION OF GROUP TABLES OF FINITE GROUPS

by Gabriel Allen, David D. Evans, and Paul Swigert
September 1969

The tables on pages 9 to 18 should be replaced with the attached tables.

NASA-Langley, 1970

Issued 4-17-70

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

07U 001 33 51 3DS 70075 00903
AIR FORCE WEAPONS LABORATORY /WLOL/
KIRTLAND AFB, [illegible]

[redacted]

AIT E. LOU BO[illegible]

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

[redacted]

TECH LIBRARY KAFB, NM
0132211

FORTRAN PROGRAM FOR MACHINE COMPUTATION OF GROUP TABLES OF FINITE GROUPS

By Gabriel Allen, David D. Evans, and Paul Swigert

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

ABSTRACT

A FORTRAN program has been written for machine computation of group tables for finite groups. The listing presented is for use on the NASA Lewis Research Center IBM 360-67 and is machine dependent. The use of the method depends on the fact that *every finite group of order n is isomorphic to some subgroup of the symmetric* group S_n. The procedure for using the program is as follows: After the proper ismorphism has been established, the appropriate elements of S_n are entered into the program as input data. The program computes and prints out the group table for these elements of S_n. Then the translation is made from the S_n elements back to the original group elements. Two examples are shown: One is the entire group S_4, and the other is A_5, the even permutation subgroup of S_5.

FORTRAN PROGRAM FOR MACHINE COMPUTATION OF GROUP TABLES OF FINITE GROUPS

by Gabriel Allen, David D. Evans, and Paul Swigert

Lewis Research Center

SUMMARY

A FORTRAN program has been written for machine computation of group tables for finite groups. The listing presented is for use on the NASA Lewis Research Center IBM 360-67 and is machine dependent. The use of the method depends on the fact that every finite group of order n is isomorphic to some subgroup of the symmetric group S_n. The procedure for using the program is as follows: After the proper isomorphism has been established, the appropriate elements of S_n are entered into the program as input data. The program computes and prints out the group table for these elements of S_n. Then the translation is made from the S_n elements back to the original group elements. Two examples are shown: One is the entire group S_4, and the other is A_5, the even permutation subgroup of S_5.

INTRODUCTION

Because of the sheer computational labor involved in constructing group tables for large finite groups, the possibility of programming such tasks for machine computation is very appealing. However, group multiplication cannot be carried out on a machine by simply inserting an * between two elements of a group. A machine program for computing group tables is actually more closely related to some of the programs recently devised for proving mathematical theorems and algorithms than to ordinary arithmetic programming (refs. 1 to 3).

All of these programs, however, require the group table to be fed in as input, after which the group can be manipulated in any of several ways to extract the desired information about its structure and properties (ref. 3). It appears that the published literature still does not contain a specific program that begins only with group elements as input and then constructs actual group tables (see ref. 4 for an up-to-date survey of com-

puters in group theory). Since such a program would be very useful to anyone working with large finite groups, a program has been devised for constructing a group table or any subsection of such a table for the symmetric group S_n. This machine-dependent program, which is written in H level FORTRAN for the NASA Lewis Research Center IBM 360-67, uses as input only the group elements in the form of cycles. As is well known, every finite group of order n is isomorphic to some subgroup of S_n (ref. 5). Therefore, a table for any finite group G can be obtained from the table of the appropriate symmetric group once the isomorphism between the elements of G and those of the subgroup of S_n has been given.

Core storage limits the size of the multiplication table which can be handled by the program. Therefore, a subroutine which stores four 8-bit words in a single 32-bit storage location is used to gain more storage. This procedure is referred to as "packing" the storage locations.

Examples of output for S_4 and A_5 (the alternating group on five letters) are presented.

DEFINITIONS AND CONVENTIONS

The entire report makes use of properties of the permutation group S_n. An element of the permutation group represents a rearrangement of a number of labeled objects among a number of labeled locations. To begin with, each object is assumed to be in the location which bears the same label as the object. The term standard configuration (SC) is used to describe that arrangement in which the ith object is in the ith location for i = 1 to N, where N is the number of objects and/or locations. When N = 3, for example, SC means

Location	1	2	3
Object	1	2	3

Every group element in S_n can be written as a product of cycles. A cycle is a shorthand notation which describes the number and nature of changes effected by a given permutation. For example, the cycle (1, 3, 2) indicates that objects 1, 2, and 3 are to be permuted in some manner and that all other objects are to remain fixed. The notation is so economical that it cannot describe the nature of the permutation without some additional convention. Even though these conventions are well known, they are described here for convenience.

If the individual numbers or letters in each cycle are called units, then a unit may signify either a location or an object. Secondly, the convention may be adopted of going from left to right or from right to left within a cycle, so that there are four choices altogether. There are only two independent choices, however, since moving objects to the left must result in the same distribution of objects among locations as moving locations to the right and vice versa. The convention used here is that in which the units denote objects and each unit is moved to the location currently occupied by the unit to its left. An example should make this clear. Consider the effect of operation (1, 3, 2). When it operates on the SC, the result is

$$(1,3,2) \times \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 1 & 2 & 3 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 3 & 1 & 2 \\ \hline \end{array}$$

Location label

Object label

When (1, 3, 2) operates on the configuration on the right side, the result is

$$(1,3,2) \times \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 3 & 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 2 & 3 & 1 \\ \hline \end{array}$$

An interchange of two units is called a transposition. A property of permutations which is very useful in programming group operations is that any permutation can be obtained by successive transpositions. For example, (1, 2, 3) = (1, 2)(2, 3). Therefore, a product of two group elements can be expressed as a single chain of transpositions. An examination of the result of this chain on the SC can then show the single group operation which would yield the same rearrangement. This group operation is then the product of the two original operations. The preceding few sentences are the heart of the method used herein.

USE OF CLASS ALGEBRA TABLES

It is clear that, in the case of large finite groups, even the group table itself is difficult to use because of the sheer physical size of the table and the inherent difficulty of accurately locating the proper entry in a given row and column. It so happens that for most groups there is a useful way of breaking up the group table into smaller blocks which provides important information about the gross structure of the group. This method is simply to break up the group into what are called class product blocks. Let K_i denote the set of group elements in the ith class of group G. Then $K_I * K_J$ is

called the $(IJ)^{th}$ class product block. Such a block is the table of group multiplication of all the elements of the Jth class used as column headings by all the elements in the Ith class used as row headings. The resulting gross structure of the group can be schematically summarized in the class algebra table (ref. 6) for the group.

A very simple example of such a breakup can be shown here by considering S_3. It has two classes in addition to the trivial class K_0, which consists entirely of the identity element E. Class K_2 is defined here as the collection of cycles of length three. Thus, K_2 consists of the two group elements (1, 2, 3) and (3, 2, 1). The collection of two-unit cycles (or transpositions) makes up the third class. Thus, K_3 consists of the elements (1, 2), (1, 3), and (2, 3). Therefore, the gross structure of the 36 entry group table for S_3 can be shown by the nine-element class algebra table:

	K_0	K_2	K_3
K_0	K_0	K_2	K_3
K_2	K_2	$2K_0 + K_2$	$2K_3$
K_3	K_3	$2K_3$	$3K_0 + 3K_2$

In examining this table, it should be noted that an entry such as $3K_0 + 3K_2$ in the class product block (33) means that, in this block, every element in class K_2 and in class K_0 appears exactly three times.

The five class product blocks $K_0 * K_0$, $K_0 * K_2$, $K_0 * K_3$, $K_2 * K_0$, and $K_3 * K_0$ consist merely of the three classes themselves. Since, for any two classes K_i and K_j, $K_i * K_j = K_j * K_i$, all the additional information about the gross structure of S_3 can be obtained from the three class product blocks $K_2 * K_2$, $K_3 * K_3$, and $K_2 * K_3$.

Examples

Before examining the output of some actual cases, it should be noted that the form in which these tables appears is slightly more efficient than the conventional group table. A conventional group table for a group of order n containing the group elements $E, g_2, \ldots, g_n$ has the form

	E	g_2	. . .	g_n
E	E	g_2	. . .	g_n
g_2	g_2	X	. . .	X
.	.	.	. . .	.
.	.	.		.
.	.	.		.
g_n	g_n	X		X

The first row and first column merely repeat the row and column headings and are, in a sense, redundant. The form of the tables in this program omits the first row and first column of the group table. Of course, the entries still "appear" on the page, but only as row or column headings.

Example 1 - S_4. - This group is small enough so that the entire group table can be printed out in a reasonable space. Therefore, the entire set of 24 group operations was entered as data and the group table constructed. The output for this group follows.

GROUP (1,2,3,4)

S4

EXAMPLE 1 - S_4

	(1,2,4):	(4,2,1):	(2,3,4):	(4,3,2):	(1,4,3):	(3,4,1):	(3,2,1):
(1,2,4):	(4,2,1)	E	(2,3,1)	(2,1)(4,3)	(4,3,2)	(3,1)(4,2)	(3,4,1)
(4,2,1):	E	(2,4,1)	(4,1)(3,2)	(4,3,1)	(2,1)(4,3)	(3,2,1)	(3,1)(4,2)
(2,3,4):	(3,4,1)	(2,1)(4,3)	(4,3,2)	E	(2,3,1)	(4,1)(3,2)	(4,2,1)
(4,3,2):	(4,1)(3,2)	(3,2,1)	E	(3,4,2)	(3,1)(4,2)	(2,4,1)	(2,1)(4,3)
(1,4,3):	(2,3,1)	(3,1)(4,2)	(4,2,1)	(4,1)(3,2)	(3,4,1)	E	(4,3,2)
(3,4,1):	(2,1)(4,3)	(3,4,2)	(3,1)(4,2)	(3,2,1)	E	(4,3,1)	(4,1)(3,2)
(3,2,1):	(4,3,2)	(4,1)(3,2)	(3,4,1)	(3,1)(4,2)	(4,2,1)	(2,1)(4,3)	(2,3,1)
(1,2,3):	(3,1)(4,2)	(4,3,1)	(2,1)(4,3)	(2,4,1)	(4,1)(3,2)	(3,4,2)	E
(1,4)(2,3):	(3,2,1)	(4,3,2)	(4,3,1)	(4,2,1)	(3,4,2)	(2,3,1)	(2,4,1)
(1,2)(3,4):	(3,4,2)	(3,4,1)	(2,4,1)	(2,3,1)	(3,2,1)	(4,2,1)	(4,3,1)
(1,3)(2,4):	(4,3,1)	(2,3,1)	(3,2,1)	(3,4,1)	(2,4,1)	(4,3,2)	(3,4,2)
(1,3,4,2):	(4,3)	(2,3,4,1)	(3,2,4,1)	(3,1)	(2,1)	(4,3,2,1)	(4,2,3,1)
(2,4,3,1):	(4,2,3,1)	(3,1)	(2,1)	(2,3,4,1)	(3,2,4,1)	(4,2)	(4,3)
(4,2,3,1):	(3,1)	(2,4,3,1)	(4,3,2,1)	(4,1)	(2,3,4,1)	(3,2)	(4,2)
(1,3,2,4):	(4,3,2,1)	(3,2)	(3,1)	(3,4,2,1)	(4,2)	(2,4,3,1)	(2,3,4,1)
(4,3,2,1):	(3,2)	(3,2,4,1)	(4,1)	(4,2,3,1)	(3,4,2,1)	(2,1)	(2,4,3,1)
(1,2,3,4):	(3,4,2,1)	(4,3)	(2,4,3,1)	(2,1)	(3,2)	(4,2,3,1)	(4,1)
(3,4):	(2,3,4,1)	(3,4,2,1)	(4,2)	(3,2)	(3,1)	(4,1)	(4,3,2,1)
(1,2):	(4,2)	(4,1)	(2,3,4,1)	(2,4,3,1)	(4,3,2,1)	(3,4,2,1)	(3,1)
(1,3):	(2,4,3,1)	(4,2,3,1)	(3,4,2,1)	(3,2,4,1)	(4,1)	(4,3)	(3,2)
(2,4):	(4,1)	(2,1)	(3,2)	(4,3)	(2,4,3,1)	(3,2,4,1)	(3,4,2,1)
(1,4):	(2,1)	(4,2)	(4,2,3,1)	(4,3,2,1)	(4,3)	(3,1)	(3,2,4,1)
(2,3):	(3,2,4,1)	(4,3,2,1)	(4,3)	(4,2)	(4,2,3,1)	(2,3,4,1)	(2,1)

	(1,2,3):	(1,4)(2,3):	(1,2)(3,4):	(1,3)(2,4):	(1,3,4,2):	(2,4,3,1):	(4,2,3,1):
(1,2,4):	(4,1)(3,2)	(3,4,2)	(4,3,1)	(3,2,1)	(3,1)	(4,3,2,1)	(3,2)
(4,2,1):	(3,4,2)	(2,3,1)	(4,3,2)	(3,4,1)	(3,2,4,1)	(4,3)	(2,3,4,1)
(2,3,4):	(3,1)(4,2)	(2,4,1)	(3,2,1)	(4,3,1)	(4,3,2,1)	(3,1)	(2,4,3,1)
(4,3,2):	(4,3,1)	(3,4,1)	(4,2,1)	(2,3,1)	(2,1)	(4,2,3,1)	(3,1)
(1,4,3):	(2,1)(4,3)	(3,2,1)	(2,4,1)	(3,4,2)	(4,2)	(2,3,4,1)	(3,4,2,1)
(3,4,1):	(2,4,1)	(4,3,2)	(2,3,1)	(4,2,1)	(4,2,3,1)	(2,1)	(4,2)
(3,2,1):	E	(4,3,1)	(3,4,2)	(2,4,1)	(2,3,4,1)	(4,2)	(4,1)
(1,2,3):	(3,2,1)	(4,2,1)	(3,4,1)	(4,3,2)	(4,3)	(3,2,4,1)	(4,3,2,1)
(1,4)(2,3):	(3,4,1)	E	(3,1)(4,2)	(2,1)(4,3)	(2,4,3,1)	(3,4,2,1)	(4,3)
(1,2)(3,4):	(4,3,2)	(3,1)(4,2)	E	(4,1)(3,2)	(4,1)	(3,2)	(3,2,4,1)
(1,3)(2,4):	(4,2,1)	(2,1)(4,3)	(4,1)(3,2)	E	(3,2)	(4,1)	(2,1)
(1,3,4,2):	(4,2)	(2,4,3,1)	(3,2)	(4,1)	E	(4,1)(3,2)	(2,4,1)
(2,4,3,1):	(4,3,2,1)	(3,4,2,1)	(4,1)	(3,2)	(4,3,2)	E	(3,2,1)
(4,2,3,1):	(3,4,2,1)	(2,1)	(3,2,4,1)	(4,3)	(2,3,1)	(4,3,2)	(4,1)(3,2)
(1,3,2,4):	(4,1)	(4,3)	(3,1)	(2,1)	(2,4,1)	(3,4,2)	(3,4,1)
(4,3,2,1):	(4,3)	(3,1)	(4,2)	(4,3,1)	(4,3,1)	(3,2,1)	(4,3,2)
(1,2,3,4):	(3,2,4,1)	(4,2)	(3,1)	(3,4,2)	(4,2,1)	(2,3,1)	(3,1)(4,2)
(3,4):	(2,4,3,1)	(4,3,2,1)	(2,1)	(4,2,3,1)	(3,4,1)	(4,3,2)	(4,2,1)
(1,2):	(3,2)	(4,2,3,1)	(4,3)	(4,1)	(3,4,2,1)	(4,2,1)	(4,3,1)
(1,3):	(2,1)	(4,3,2,1)	(4,2,3,1)	(3,2,4,1)	(4,2,3,1)	(3,4,1)	(4,2,3,1)
(2,4):	(4,2,3,1)	(2,3,4,1)	(4,3,2,1)	(4,2)	(3,4,2,1)	(3,4,2)	(2,4,1)
(1,4):	(2,3,4,1)	(3,2)	(2,4,3,1)	(3,1)	(3,1)(4,2)	(2,4,1)	(3,4,2)
(2,3):	(3,1)	(4,1)	(3,4,2,1)	(2,4,3,1)	(2,1)(4,3)	(3,1)(4,2)	(4,3,1)

	(1,3,2,4):	(4,3,2,1):	(1,2,3,4):	(3,4):	(1,2):	(1,3):	(2,4):
(1,2,4):	(3,4,2,1)	(4,3)	(4,2,3,1)	(2,4,3,1)	(4,1)	(3,2,4,1)	(2,1)
(4,2,1):	(3,1)	(2,4,3,1)	(3,2)	(4,3,2,1)	(4,2)	(3,4,2,1)	(4,1)
(2,3,4):	(4,1)	(2,1)	(3,2,4,1)	(3,2)	(3,4,2,1)	(4,2,3,1)	(4,3)
(4,3,2):	(2,3,4,1)	(3,4,2,1)	(4,1)	(4,2)	(4,3,2,1)	(2,4,3,1)	(3,2)
(1,4,3):	(3,2)	(3,2,4,1)	(2,1)	(4,1)	(2,4,3,1)	(4,3)	(4,2,3,1)
(3,4,1):	(4,3,2,1)	(3,2)	(2,4,3,1)	(3,1)	(2,3,4,1)	(4,1)	(3,4,2,1)
(3,2,1):	(2,4,3,1)	(4,2,3,1)	(4,3)	(3,4,2,1)	(3,2)	(2,1)	(3,2,4,1)
(1,2,3):	(4,2)	(4,1)	(3,4,2,1)	(2,3,4,1)	(3,1)	(3,2)	(2,4,3,1)
(1,4)(2,3):	(2,1)	(4,2)	(3,1)	(4,2,3,1)	(3,2,4,1)	(2,3,4,1)	(4,3,2,1)
(1,2)(3,4):	(4,2,3,1)	(3,1)	(4,2)	(2,1)	(4,3)	(4,3,2,1)	(2,3,4,1)
(1,3)(2,4):	(4,3)	(2,3,4,1)	(4,3,2,1)	(3,2,4,1)	(4,2,3,1)	(4,2)	(3,1)
(1,3,4,2):	(4,3,1)	(2,3,1)	(4,3,2)	(3,2,1)	(3,4,2)	(4,2,1)	(3,4,1)
(2,4,3,1):	(3,4,2)	(3,4,1)	(4,2,1)	(2,4,1)	(4,3,1)	(4,3,2)	(2,3,1)
(4,2,3,1):	F	(2,4,1)	(3,2,1)	(4,1)(3,2)	(3,1)(4,2)	(3,4,2)	(4,3,1)
(1,3,2,4):	(2,1)(4,3)	(3,4,2)	(4,3,1)	(3,1)(4,2)	(4,1)(3,2)	(2,4,1)	(3,2,1)
(4,3,2,1):	(2,3,1)	(3,1)(4,2)	E	(4,2,1)	(4,3,2)	(2,1)(4,3)	(4,1)(3,2)
(1,2,3,4):	(4,2,1)	E	(3,1)(4,2)	(2,3,1)	(3,4,1)	(4,1)(3,2)	(2,1)(4,3)
(3,4):	(4,1)(3,2)	(3,2,1)	(2,4,1)	E	(2,1)(4,3)	(4,3,1)	(3,4,2)
(1,2):	(3,1)(4,2)	(4,3,1)	(3,4,2)	(2,1)(4,3)	E	(3,2,1)	(2,4,1)
(1,3):	(4,3,2)	(4,1)(3,2)	(2,1)(4,3)	(3,4,1)	(2,3,1)	E	(3,1)(4,2)
(2,4):	(3,4,1)	(2,1)(4,3)	(4,1)(3,2)	(4,3,2)	(4,2,1)	(3,1)(4,2)	E
(1,4):	(3,2,1)	(4,3,2)	(2,3,1)	(4,3,1)	(2,4,1)	(3,4,1)	(4,2,1)
(2,3):	(2,4,1)	(4,2,1)	(3,4,1)	(3,4,2)	(3,2,1)	(2,3,1)	(4,3,2)

	(1,4):	(2,3):
(1,2,4):	(4,2)	(2,3,4,1)
(4,2,1):	(2,1)	(4,2,3,1)
(2,3,4):	(2,3,4,1)	(4,2)
(4,3,2):	(3,2,4,1)	(4,3)
(1,4,3):	(3,1)	(4,3,2,1)
(3,4,1):	(4,3)	(3,2,4,1)
(3,2,1):	(4,3,2,1)	(3,1)
(1,2,3):	(4,2,3,1)	(2,1)
(1,4)(2,3):	(3,2)	(4,1)
(1,2)(3,4):	(3,4,2,1)	(2,4,3,1)
(1,3)(2,4):	(2,4,3,1)	(3,4,2,1)
(1,3,4,2):	(2,1)(4,3)	(3,1)(4,2)
(2,4,3,1):	(3,1)(4,2)	(2,1)(4,3)
(4,2,3,1):	(2,3,1)	(4,2,1)
(1,3,2,4):	(4,3,2)	(3,4,1)
(4,3,2,1):	(3,2,1)	(4,3,1)
(1,2,3,4):	(4,1)	(3,1)
(3,4):	(2,1)	(4,1)
(1,2):	(4,1)	(3,1)
(1,3):	(3,2)	(4,2)
(2,4):	(3,1)	(4,3)
(1,4):	E	(4,1)(3,2)
(2,3):	(4,1)(3,2)	E

Example 2 - A_5. - The group A_5 is the group of even permutations of S_5. It is, therefore, a proper subgroup of S_5. There are five classes of A_5 including K_0. The elements of S_5 which are expressible as five-unit cycles fall into two distinct classes of 12 group elements each. These will be called K_2 and K_3. The 20 elements expressible as three unit cycles form a class called K_4, and the 15 elements which may be expressed as products of two independent transpositions (e.g., (12) (35)) are in the class K_5. One should not be confused by the fact that the 24 five unit cycles of S_5 (which fall into a single class of S_5) fall into two distinct classes in A_5. It requires conjugation by an odd permutation of S_5 to take an element of K_2 into K_3 and vice versa. The only group elements available for conjugation in A_5 are the even permutations of S_5, and no conjugation of any element in K_2 by such an even permutation can take this element into K_3.

Since A_5 is of order 60, it is a good choice for breaking up the group table into class product blocks. The program can readily handle such blocks. However, class products do not have the closure property and elements can arise which are not included in the input for either class in the class product block. Therefore, this example serves to illustrate the warning that IDENT (see program description) has to be made long enough to accommodate the longest chain of changes in locations which any element of the full group can cause. Note, for instance, that whereas the class K_4 consists of cycles three units long, the class product block for $K_4 * K_4$ contains cycles five units long. The following class algebra table is not readily available in the literature and was obtained from the class product blocks computed by this program:

K_0	K_2	K_3	K_4	K_5
K_2	$12K_0 + 5K_2 + K_3 + 3K_4$	$K_2 + K_3 + 3K_4 + 4K_5$	$5K_2 + 5K_3 + 3K_4 + 4K_5$	$5K_3 + 3K_4 + 4K_5$
K_3	$K_2 + K_3 + 3K_4 + 4K_5$	$12K_0 + K_2 + 5K_3 + 3K_4$	$5K_2 + 5K_3 + 3K_4 + 4K_5$	$5K_2 + 3K_4 + 4K_5$
K_4	$5K_2 + 5K_3 + 3K_4 + 4K_5$	$5K_2 + 5K_3 + 3K_4 + 4K_5$	$20K_0 + 5K_2 + 5K_3 + 7K_4 + 8K_5$	$5K_2 + 5K_3 + 6K_4 + 4K_5$
K_5	$5K_3 + 3K_4 + 4K_5$	$5K_2 + 3K_4 + 4K_5$	$5K_2 + 5K_3 + 6K_4 + 4K_5$	$15K_0 + 5K_2 + 5K_3 + 3K_4 + 2K_5$

Of course, the class algebra table only gives gross information about the structure of the group. For example, although $K_2 * K_4 = K_4 * K_2$, both class product blocks are needed since one cannot predict the details of block $K_2 * K_4$ from $K_4 * K_2$ or vice versa. Therefore, all 16 nontrivial class product blocks were obtained and are shown in the following pages.

EXAMPLE 2 - A_5

GROUP (1,2,3,4,5)
K2*K2

	(1,2,3,4,5):	(1,2,5,3,4):	(1,4,5,2,3):	(1,4,2,3,5):	(1,3,5,4,2):	(1,3,4,2,5):	(5,4,3,2,1):
(1,2,3,4,5):	(3,5,2,4,1)	(3,5,4,2,1)	(5,3,2,4,1)	(5,2,4,3,1)	(4,3,1)	(4,3,5,2,1)	E
(1,2,5,3,4):	(5,2,4,3,1)	(5,4,2,3,1)	(4,3,2)	(4,5,2)	(4,5,1)	(4,5,2,3,1)	(3,5,1)
(1,4,5,2,3):	(3,5,4,2,1)	(3,5,1)	(5,3,4,2,1)	(5,4,3,2,1)	(4,3,2)	(5,4,3)	(2,4,1)
(1,4,2,3,5):	(3,2,5,4,1)	(3,2,1)	(2,5,3,4,1)	(2,5,4,3,1)	(5,2,4,3,1)	(5,4,3,2,1)	(4,5,2)
(1,1,5,4,2):	(5,4,2)	(4,3,2)	(2,5,1)	(2,5,3,4,1)	(5,2,3,4,1)	(5,3,2,4,1)	(4,5,2,3,1)
(1,3,4,2,5):	(5,3,2,4,1)	(5,4,3,2,1)	(2,4,1)	(2,4,5,3,1)	(4,5,2,3,1)	(4,5,3,2,1)	(3,5,2)
(5,4,3,2,1):	E	(4,5,2)	(3,5,1)	(3,4,1)	(2,5,3,4,1)	(2,4,1)	(4,2,5,3,1)
(5,2,1,4,3):	(5,4,2)	E	(3,4,2,5,1)	(3,2,5,4,1)	(5,3,2,4,1)	(5,4,1)	(2,4,5,3,1)
(5,4,1,3,2):	(5,3,1)	(5,2,4,3,1)	E	(4,5,3)	(2,3,4,5,1)	(2,4,5,3,1)	(4,2,3,5,1)
(5,3,2,4,1):	(4,3,1)	(4,5,2,3,1)	(5,4,3)	E	(2,5,1)	(2,3,1)	(3,4,2,5,1)
(5,3,1,2,4):	(4,3,5,2,1)	(4,2,3,5,1)	(5,4,3,2,1)	(5,2,1)	E	(3,5,2)	(3,4,1)
(5,2,4,3,1):	(4,2,1)	(4,5,1)	(3,5,4,2,1)	(3,2,1)	(5,3,2)	E	(2,5,3,4,1)

	(5,2,1,4,3):	(5,4,1,3,2):	(5,3,2,4,1):	(5,3,1,2,4):	(5,2,4,3,1):
(1,2,3,4,5):	(5,3,1)	(4,2,1)	(5,4,2)	(3,2,5,4,1)	(5,3,2)
(1,2,5,3,4):	E	(4,2,3,5,1)	(3,5,4,2,1)	(5,4,3,2,1)	(3,2,1)
(1,4,5,2,3):	(5,3,2,4,1)	E	(2,5,1)	(3,4,2,5,1)	(2,5,3,4,1)
(1,4,2,3,5):	(2,4,5,3,1)	(5,2,1)	E	(3,4,1)	(4,5,3)
(1,3,5,4,2):	(2,3,4,5,1)	(5,2,4,3,1)	(4,3,1)	E	E
(1,3,4,2,5):	(2,3,1)	(4,3,5,2,1)	(5,4,3)	(5,4,1)	(4,2,3,5,1)
(5,4,3,2,1):	(3,4,2,5,1)	(2,4,5,3,1)	(4,5,2,3,1)	(3,5,2)	(2,3,4,5,1)
(5,2,1,4,3):	(3,2,4,5,1)	(5,3,1)	(2,3,1)	(3,4,2)	(4,2,1)
(5,4,1,3,2):	(3,4,2)	(2,4,3,5,1)	(4,3,5,2,1)	(5,2,1)	(3,5,4,2,1)
(5,3,2,4,1):	(5,4,2)	(2,3,4,5,1)	(3,4,5,2,1)	(4,3,5,2,1)	(3,2,5,4,1)
(5,2,1,4,3):	(2,1)(4,3)	(4,5,1)	(4,5,2,3,1)	(5,1)(4,2)	(4,3,1)
(5,1,2,4,3):	(5,4,2,3,1)	(5,3,2)	E	(4,5,3,2,1)	(5,4,2)
(5,2,1,3,4):	(4,3,2,5,1)	(2,4,1)	(2,3,5,4,1)	E	(2,4,3,5,1)
(5,1,4,3,2):	(2,3,1)	(4,5,3)	(4,5,2)	(5,3,4,2,1)	E
(5,4,1,2,3):	(5,3,4,2,1)	(4,3,2,5,1)	(4,3,1)	(3,2,4,5,1)	(4,2,5,3,1)

GROUP (1,2,3,4,5)
K3*K3

	(1,3,5,2,4):	(1,5,4,2,3):	(1,5,3,4,2):	(1,2,5,4,3):	(1,5,2,3,4):	(1,4,5,3,2):	(5,3,1,4,2):
(1,3,5,2,4):	(5,4,3,2,1)	(2,5,1)	(2,3,1)	(4,5,1)	(2,5,4,3,1)	(3,4,2)	E
(1,5,4,2,3):	(4,5,3)	(4,3,5,2,1)	(4,3,2,5,1)	(3,5,2,4,1)	(4,5,3,2,1)	(2,5,1)	(2,4,3,5,1)
(1,5,3,4,2):	(4,5,1)	(3,5,2,4,1)	(3,2,5,4,1)	(3,5,2)	(3,2,4,5,1)	(2,5,4,3,1)	(2,3,5,4,1)
(1,2,5,4,3):	(3,4,2)	(4,5,3,2,1)	(4,5,1)	(5,3,2,4,1)	(4,2,1)	(3,5,1)	(3,2,4,5,1)
(1,5,2,3,4):	(4,5,3,2,1)	(2,4,3,5,1)	(2,5,4,3,1)	(3,5,1)	(2,4,5,3,1)	(5,4,2)	(5,4,3)
(1,4,5,3,2):	(2,5,1)	(3,4,1)	(3,5,2,4,1)	(3,4,2)	(3,5,1)	(5,2,4,3,1)	(5,2,3,4,1)
(5,3,1,4,2):	E	(3,4,5,2,1)	(3,2,4,5,1)	(5,2,3,4,1)	(3,2,1)	(2,4,3,5,1)	(2,3,4,5,1)
(5,4,1,3,2):	(2,1)(4,3)	(4,5,1)	(4,5,2,3,1)	(5,1)(4,2)	(4,3,1)	(3,5,2)	(4,5,2)
(5,1,2,4,3):	(5,4,2,3,1)	(5,3,2)	E	(4,5,3,2,1)	(5,4,2)	(3,4,1)	(3,2,1)
(5,2,1,3,4):	(4,3,2,5,1)	(2,4,1)	(2,3,5,4,1)	E	(2,4,3,5,1)	(5,4,2,3,1)	(5,4,1)
(5,1,4,3,2):	(2,3,1)	(4,5,3)	(4,5,2)	(5,3,4,2,1)	E	(3,5,2,4,1)	(3,4,5,2,1)
(5,4,1,2,3):	(5,3,4,2,1)	(4,3,2,5,1)	(4,3,1)	(3,2,4,5,1)	(4,2,5,3,1)	E	(4,3,2)

	(5,4,1,3,2):	(5,1,2,4,3):	(5,2,1,3,4):	(5,1,4,3,2):	(5,4,1,2,3):
(1,3,5,2,4):	(5,1)(4,3)	(4,5,3,2,1)	(5,4,2,3,1)	(4,5,3)	(4,3,2,5,1)
(1,5,4,2,3):	(4,5,2)	(3,4,1)	(5,3,2)	(2,4,1)	(3,4,5,2,1)
(1,5,3,4,2):	(4,5,2,3,1)	E	(4,3,2,5,1)	(2,3,1)	(4,5,2)
(1,2,5,4,3):	(4,2)(5,3)	(5,2,3,4,1)	E	(3,5,2,4,1)	(5,3,4,2,1)
(1,5,2,3,4):	(4,5,1)	(3,2,1)	(4,2,5,3,1)	E	(3,2,4,5,1)
(1,4,5,3,2):	(2,3,1)	(5,4,2)	(2,4,3,5,1)	(5,4,2,3,1)	E
(5,3,1,4,2):	(3,5,2)	(5,4,1)	(4,3,2)	(2,3,5,4,1)	(5,2,1)
(5,4,1,3,2):	(2,4,3,5,1)	(5,3,4,2,1)	(2,3,1)	(4,2)(5,3)	(5,1)(4,3)
(5,1,2,4,3):	(4,3,1)	(4,5,2,3,1)	(4,3,5,2,1)	(3,4,5,2,1)	(4,2,5,3,1)
(5,2,1,3,4):	(4,3,1)	(5,3,2)	(4,2,3,5,1)	(5,3,1)	(4,3,2)
(5,4,1,3,2):	(2,1)(5,3)	(5,4,2,3,1)	(2,4,1)	(3,5,4,2,1)	(5,3,1)
(5,4,1,2,3):	(5,1)(4,2)	(3,4,5,2,1)	(5,3,1)	(4,5,2)	(3,4,2,5,1)

GROUP (1,2,3,4,5)
K4*K4

	(1,2,3):	(2,3,4):	(1,3,4):	(1,2,4):	(1,2,5):	(1,3,5):	(2,3,5):
(1,2,3):	(3,2,1)	(2,1)(4,3)	(3,4,2)	(3,1)(4,2)	(3,1)(5,2)	(3,5,2)	(2,1)(5,3)
(2,3,4):	(3,1)(4,2)	(4,3,2)	(4,1)(3,2)	(3,4,1)	(3,4,2,5,1)	(4,2,3,5,1)	(4,2)(5,3)
(1,3,4):	(2,4,1)	(3,1)(4,2)	(4,3,1)	(2,1)(4,3)	(2,5,3,4,1)	(4,1)(5,3)	(3,5,2,4,1)
(1,2,4):	(4,1)(3,2)	(2,3,1)	(3,1)(4,2)	(4,2,1)	(4,1)(5,2)	(3,5,2,4,1)	(2,3,5,4,1)
(1,2,5):	(5,1)(3,2)	(2,3,4,5,1)	(3,4,2,5,1)	(5,1)(4,2)	(5,2,1)	(3,1)(5,2)	(2,3,1)
(1,3,5):	(2,5,1)	(3,4,2,5,1)	(5,1)(4,3)	(2,4,3,5,1)	(2,1)(5,3)	(5,3,1)	(3,1)(5,2)
(2,3,5):	(3,1)(5,2)	(5,2)(4,3)	(5,2,3,4,1)	(3,5,2,4,1)	(3,5,1)	(5,1)(3,2)	(5,3,2)
(1,4,5):	(2,3,4,5,1)	(4,2,3,5,1)	(3,5,1)	(2,5,1)	(2,1)(5,4)	(3,1)(5,4)	(4,5,2,3,1)
(2,4,5):	(4,5,2,3,1)	(3,5,2)	(3,5,2,4,1)	(4,1)(5,2)	(4,5,1)	(3,2,4,5,1)	(3,2)(5,4)
(3,4,5):	(2,4,5,3,1)	(4,2)(5,3)	(4,1)(5,3)	(2,5,3,4,1)	(4,5,1)	(4,5,1)	(4,5,2)
(3,2,1):	E	(3,4,1)	(2,1)(4,3)	(4,3,2)	(5,3,2)	(2,1)(5,3)	(3,5,1)
(4,3,2):	(4,3,1)	E	(2,4,1)	(4,1)(3,2)	(4,3,2,5,1)	(2,4,3,5,1)	(5,4,3)
(4,3,1):	(2,1)(4,3)	(4,2,1)	E	(2,3,1)	(2,5,4,3,1)	(5,4,3)	(4,3,5,2,1)
(4,2,1):	(3,4,2)	(4,1)(3,2)	(3,2,1)	E	(5,4,2)	(3,5,4,2,1)	(4,2,3,5,1)
(5,2,1):	(3,5,2)	(5,2,3,4,1)	(3,4,5,2,1)	(4,5,2)	E	(3,2,1)	(5,1)(3,2)
(5,3,1):	(2,1)(5,3)	(5,3,4,2,1)	(4,5,3)	(2,4,5,3,1)	(2,3,1)	E	(5,2,1)
(5,3,2):	(5,3,1)	(4,5,3)	(2,5,3,4,1)	(5,3,2,4,1)	(5,1)(3,2)	(2,5,1)	E
(5,4,1):	(2,3,5,4,1)	(5,4,2,3,1)	(3,1)(5,4)	(2,1)(5,4)	(2,4,1)	(3,4,1)	(5,2,3,4,1)
(5,4,2):	(5,4,2,3,1)	(3,2)(5,4)	(3,2,5,4,1)	(5,4,1)	(5,1)(4,2)	(3,4,2,5,1)	(3,4,2)
(5,4,3):	(2,5,4,3,1)	(5,4,2)	(5,4,1)	(2,3,5,4,1)	(2,4,3,5,1)	(5,1)(4,3)	(5,2)(4,3)

	(1,4,5):	(2,4,5):	(3,4,5):	(3,2,1):	(4,3,2):	(4,3,1):	(4,2,1):
(1,2,3):	(4,5,2,3,1)	(2,4,5,3,1)	(2,3,4,5,1)	E	(2,4,1)	(4,1)(3,2)	(4,3,1)
(2,3,4):	(2,3,4,5,1)	(4,5,3)	(3,2)(5,4)	(4,2,1)	E	(2,3,1)	(2,1)(4,3)
(1,3,4):	(4,5,3)	(3,4,5,2,1)	(3,1)(5,4)	(4,1)(3,2)	(3,2,1)	E	(3,4,2)
(1,2,4):	(4,5,2)	(2,1)(5,4)	(2,4,5,3,1)	(3,4,1)	(2,1)(4,3)	(4,3,2)	E
(1,2,5):	(4,1)(5,2)	(2,4,1)	(2,5,3,4,1)	(3,5,1)	(2,4,3,5,1)	(4,3,2,5,1)	(4,5,1)
(1,3,5):	(4,1)(5,3)	(3,5,2,4,1)	(3,4,1)	(5,1)(3,2)	(3,2,4,5,1)	(4,5,1)	(4,2,3,5,1)
(2,3,5):	(4,2,3,5,1)	(4,2)(5,3)	(3,4,2)	(5,2,1)	(4,5,2)	(4,5,2,3,1)	(4,3,5,2,1)
(1,4,5):	(5,4,1)	(4,1)(5,2)	(4,1)(5,3)	(3,2,4,5,1)	(4,3,2,5,1)	(5,1)(4,3)	(5,1)(4,2)
(2,4,5):	(5,1)(4,2)	(5,4,2)	(4,2)(5,3)	(3,4,5,2,1)	(5,2)(4,3)	(5,2,4,3,1)	(5,2,1)
(3,4,5):	(5,1)(4,3)	(5,2)(4,3)	(5,4,3)	(4,5,3,2,1)	(5,3,2)	(5,3,1)	(5,3,4,2,1)
(3,2,1):	(4,5,3,2,1)	(3,2,4,5,1)	(3,4,5,2,1)	(2,3,1)	(3,1)(4,2)	(4,2,1)	(4,1)(3,2)
(4,3,2):	(3,2,4,5,1)	(3,2)(5,4)	(4,5,2)	(2,1)(4,3)	(3,4,2)	(3,1)(4,2)	(3,2,1)
(4,3,1):	(3,1)(5,4)	(4,5,2,3,1)	(4,5,1)	(4,3,2)	(4,1)(3,2)	(3,4,1)	(3,1)(4,2)
(4,2,1):	(2,1)(5,4)	(4,5,1)	(4,5,3,2,1)	(3,1)(4,2)	(4,3,1)	(2,1)(4,3)	(2,4,1)
(5,2,1):	(4,2,1)	(5,1)(4,2)	(5,3,4,2,1)	(3,1)(5,2)	(5,2,4,3,1)	(4,3,5,2,1)	(4,1)(5,2)
(5,3,1):	(4,3,1)	(5,2,4,3,1)	(5,1)(4,3)	(5,3,2)	(5,3,2,4,1)	(4,1)(5,3)	(4,2,5,3,1)
(5,3,2):	(4,3,2,5,1)	(4,3,2)	(5,2)(4,3)	(2,1)(5,3)	(4,2)(5,3)	(4,2,1)	(4,1,1)(3,2)
(5,4,1):	F	(5,2,1)	(5,3,1)	(3,2,5,4,1)	(5,4,3,2,1)	(5,4,3)	(5,4,2)
(5,4,2):	(2,5,1)	E	(5,3,2)	(3,5,4,2,1)	(5,4,3)	(2,5,4,3,1)	(2,1)(5,4)
(5,4,3):	(3,5,1)	(3,5,2)	E	(5,4,3,2,1)	(3,2)(5,4)	(3,1)(5,4)	(3,5,4,2,1)

	(5,2,1):	(5,3,1):	(5,3,2):	(5,4,1):	(5,4,2):	(5,4,3):
(1,2,3):	(5,3,1)	(5,1)(3,2)	(2,5,1)	(5,4,2,3,1)	(2,5,4,3,1)	(2,3,5,4,1)
(2,3,4):	(5,3,4,2,1)	(5,4,2,3,1)	(5,4,2)	(5,2,3,4,1)	(5,2)(4,3)	(3,5,2)
(1,3,4):	(5,2,3,4,1)	(5,4,1)	(3,2,5,4,1)	(5,1)(4,3)	(3,4,2,5,1)	(3,5,1)
(1,2,4):	(5,4,1)	(5,3,2,4,1)	(2,5,3,4,1)	(5,1)(4,2)	(2,5,1)	(2,4,3,5,1)
(1,2,5):	E	(5,3,2)	(2,1)(5,3)	(5,4,2)	(2,1)(5,4)	(2,5,4,3,1)
(1,3,5):	(3,5,2)	F	(3,2,1)	(5,4,3)	(3,5,4,2,1)	(3,1)(5,4)
(2,3,5):	(2,1)(5,3)	(2,3,1)	E	(2,3,5,4,1)	(5,4,3)	(3,2)(5,4)
(1,4,5):	(4,5,2)	(4,5,3)	(4,5,3,2,1)	E	(4,2,1)	(4,3,1)
(2,4,5):	(2,1)(5,4)	(2,4,5,3,1)	(4,5,3)	(2,4,1)	E	(4,3,2)
(3,4,5):	(3,1)(5,4)	(3,1)(5,4)	(3,1)(5,2)	(3,4,1)	(3,4,2,5,1)	E
(3,2,1):	(5,1)(4,2)	(5,2,1)	(4,3,2,5,1)	(5,3,2,4,1)	(5,3,2,4,1)	(4,5,1)
(4,3,2):	(5,4,2,3,1)	(5,2,4,3,1)	(5,3,2,4,1)	(5,3,1)	(4,2,5,3,1)	(4,1)(5,3)
(4,3,1):	(5,4,3,2,1)	(5,3,4,2,1)	(4,2,5,3,1)	(5,2,1)	(4,1)(5,2)	(4,3,5,2,1)
(4,2,1):	(2,5,1)	(2,1)(5,3)	(5,3,1)	(2,1)(5,4)	(5,4,1)	(5,4,3,2,1)
(5,2,1):	(3,1)(5,2)	(3,5,1)	(5,1)(3,2)	(3,1)(5,4)	(5,4,2,3,1)	(5,4,2)
(5,3,1):	(3,2,1)	(3,1)(5,2)	(3,5,2)	(3,2,5,4,1)	(5,3,2,4,1)	(4,2)(5,3)
(5,3,2):	(5,4,1)	(5,2,4,3,1)	(5,2)(4,3)	(4,5,1)	(5,4,3,2,1)	(4,1)(5,3)
(5,4,1):	(5,2,1)	(5,3,4,2,1)	(4,3,2,5,1)	(5,3,1)	(4,2,5,3,1)	(4,3,5,2,1)
(5,4,2):	(4,2,1)	(2,1)(5,3)	(4,2,5,3,1)	(5,2,1)	(4,1)(5,2)	(5,4,3,2,1)
(5,4,3):	(4,3,5,2,1)	(3,5,1)	(4,3,2)	(2,1)(5,4)	(5,4,1)	(5,4,2)
		(3,1)(5,2)		(3,1)(5,4)	(5,4,2,3,1)	(5,4,1)
		(4,1)(5,3)		(3,2,5,4,1)	(5,3,2,4,1)	(5,4,2)
		(4,2,5,3,1)		(4,5,1)	(5,1)(4,3)	(5,1)(4,3)
		(4,3,1)		(4,1)(5,2)	(4,2)(5,3)	(4,5,3)
				(4,1)(5,3)		

GROUP (1,2,3,4,5)
K5*K5

	(1,2)(3,4):	(1,3)(2,4):	(1,4)(2,3):	(1,2)(3,5):	(1,3)(2,5):	(1,5)(2,3):	(1,2)(4,5):
(1,2)(3,4):	E	(4,1)(3,2)	(3,1)(4,2)	(5,4,3)	(4,3,2,5,1)	(5,2,4,3,1)	(4,5,3)
(1,3)(2,4):	(4,1)(3,2)	E	(2,1)(4,3)	(4,2,3,5,1)	(5,4,2)	(5,3,4,2,1)	(4,5,2,3,1)
(1,4)(2,3):	(3,1)(4,2)	(2,1)(4,3)	E	(3,5,2,4,1)	(2,5,3,4,1)	(5,4,1)	(3,2,4,5,1)
(1,2)(3,5):	(4,5,3)	(5,3,2,4,1)	(4,2,5,3,1)	E	(5,1)(3,2)	(3,1)(5,2)	(5,4,3)
(1,3)(2,5):	(5,2,3,4,1)	(4,5,2)	(4,3,5,2,1)	(5,1)(3,2)	E	(2,1)(5,3)	(5,4,2,3,1)
(1,5)(2,3):	(3,4,2,5,1)	(2,4,3,5,1)	(4,5,1)	(3,1)(5,2)	(2,1)(5,3)	E	(3,2,5,4,1)
(1,2)(4,5):	(5,4,3)	(3,2,5,4,1)	(5,4,2,3,1)	(4,5,3)	(3,2,4,5,1)	(4,5,2,3,1)	E
(1,4)(2,5):	(5,2,4,3,1)	(3,4,5,2,1)	(3,5,2)	(5,3,2,4,1)	(3,4,1)	(2,3,5,4,1)	(5,1)(4,2)
(1,5)(2,4):	(4,3,2,5,1)	(3,5,1)	(2,3,4,5,1)	(4,2,5,3,1)	(3,5,4,2,1)	(3,4,2)	(4,1)(5,2)
(1,3)(4,5):	(2,3,5,4,1)	(5,4,2)	(5,4,3,2,1)	(2,3,4,5,1)	(4,5,2)	(4,5,3,2,1)	(2,3,1)
(1,4)(3,5):	(2,4,5,3,1)	(5,3,4,2,1)	(5,3,2)	(2,4,1)	(5,2,3,4,1)	(3,2,5,4,1)	(2,4,3,5,1)
(1,5)(3,4):	(2,5,1)	(4,2,3,5,1)	(3,2,4,5,1)	(2,5,4,3,1)	(4,3,5,2,1)	(4,3,2)	(2,5,3,4,1)
(2,3)(4,5):	(3,5,4,2,1)	(2,5,4,3,1)	(5,4,1)	(3,4,5,2,1)	(2,4,5,3,1)	(4,5,1)	(3,2,1)
(2,4)(3,5):	(4,5,3,2,1)	(5,3,1)	(2,5,3,4,1)	(4,2,1)	(5,4,2,3,1)	(3,4,2,5,1)	(4,3,5,2,1)
(2,5)(3,4):	(5,2,1)	(4,5,2,3,1)	(3,5,2,4,1)	(5,4,3,2,1)	(4,3,1)	(2,4,3,5,1)	(5,3,4,2,1)

	(1,4)(2,5):	(1,5)(2,4):	(1,3)(4,5):	(1,4)(3,5):	(1,5)(3,4):	(2,3)(4,5):	(2,4)(3,5):
(1,2)(3,4):	(3,4,2,5,1)	(5,2,3,4,1)	(4,5,3,2,1)	(3,5,4,2,1)	(5,2,1)	(2,4,5,3,1)	(2,3,5,4,1)
(1,3)(2,4):	(2,5,4,3,1)	(5,3,1)	(4,5,2)	(2,4,3,5,1)	(5,3,2,4,1)	(3,4,5,2,1)	(3,5,1)
(1,4)(2,3):	(5,3,2)	(5,4,3,2,1)	(2,3,4,5,1)	(3,5,2)	(5,4,2,3,1)	(4,5,1)	(4,3,5,2,1)
(1,2)(3,5):	(4,2,3,5,1)	(3,5,2,4,1)	(5,4,3,2,1)	(4,2,1)	(3,4,5,2,1)	(2,5,4,3,1)	(2,4,1)
(1,3)(2,5):	(4,3,1)	(2,4,5,3,1)	(5,4,2)	(4,3,2,5,1)	(2,5,3,4,1)	(3,5,4,2,1)	(3,2,4,5,1)
(1,5)(2,3):	(4,5,3,2,1)	(4,3,2)	(2,3,5,4,1)	(4,5,2,3,1)	(3,4,2)	(5,4,1)	(5,2,4,3,1)
(1,2)(4,5):	(5,1)(4,2)	(4,1)(5,2)	(3,2,1)	(5,3,4,2,1)	(4,3,5,2,1)	(2,3,1)	(2,5,3,4,1)
(1,4)(2,5):	F	(2,1)(5,4)	(3,4,2,5,1)	(5,3,2)	(2,5,4,3,1)	(4,2,3,5,1)	(4,5,3,2,1)
(1,5)(2,4):	(2,1)(5,4)	E	(3,5,2,4,1)	(2,4,5,3,1)	(4,3,2)	(5,2,3,4,1)	(5,3,1)
(1,3)(4,5):	(5,2,4,3,1)	(4,2,5,3,1)	E	(5,1)(4,3)	(4,1)(5,3)	(3,2,1)	(3,4,2,5,1)
(1,4)(3,5):	(3,5,2)	(3,5,4,2,1)	(5,1)(4,3)	E	(3,1)(5,4)	(4,3,2,5,1)	(4,2,1)
(1,5)(3,4):	(3,4,5,2,1)	(3,4,2)	(4,1)(5,3)	(3,1)(5,4)	E	(5,3,2,4,1)	(5,4,2,3,1)
(2,3)(4,5):	(5,3,2,4,1)	(4,3,2,5,1)	(2,3,1)	(5,2,3,4,1)	(4,2,3,5,1)	E	(5,2)(4,3)
(2,4)(3,5):	(2,3,5,4,1)	(3,5,1)	(5,2,4,3,1)	(2,4,1)	(3,2,4,5,1)	(5,2)(4,3)	E
(2,5)(3,4):	(3,4,1)	(2,3,4,5,1)	(4,2,5,3,1)	(3,2,5,4,1)	(2,5,1)	(4,2)(5,3)	(3,2)(5,4)

	(2,5)(3,4):
(1,2)(3,4):	(2,5,1)
(1,3)(2,4):	(3,2,5,4,1)
(1,4)(2,3):	(4,2,5,3,1)
(1,2)(3,5):	(2,3,4,5,1)
(1,3)(2,5):	(3,4,1)
(1,5)(2,3):	(5,3,4,2,1)
(1,2)(4,5):	(2,4,3,5,1)
(1,4)(2,5):	(4,3,1)
(1,5)(2,4):	(5,4,3,2,1)
(1,3)(4,5):	(3,5,2,4,1)
(1,4)(3,5):	(4,5,2,3,1)
(1,5)(3,4):	(5,2,1)
(2,3)(4,5):	(4,2)(5,3)
(2,4)(3,5):	(3,2)(5,4)
(2,5)(3,4):	F

GROUP (1,2,3,4,5)
K2*K3

	(1,3,5,2,4):	(1,5,4,2,3):	(1,5,3,4,2):	(1,2,5,4,3):	(1,5,2,3,4):	(1,4,5,3,2):	(5,3,1,4,2):
(1,2,3,4,5):	(4,2,5,3,1)	(4,3,2)	(5,4,3)	(3,2,1)	(4,2)(5,3)	(5,4,1)	(5,4,3,2,1)
(1,2,5,3,4):	(4,2,1)	(3,2,4,5,1)	(3,1)(5,4)	(5,1)(3,2)	(3,1)(4,2)	(5,4,3)	(3,2)(5,4)
(1,4,5,2,3):	(5,3,2)	(2,1)(4,3)	(2,4,3,5,1)	(3,4,1)	(2,1)(5,3)	(5,1)(4,2)	(5,1)(4,3)
(1,4,2,3,5):	(5,3,1)	(5,2)(4,3)	(4,3,2)	(3,4,5,2,1)	(5,3,2)	(2,4,1)	(2,1)(4,3)
(1,3,5,4,2):	(5,1)(4,3)	(4,1)(5,2)	(4,1)(3,2)	(4,5,2)	(4,3,2,5,1)	(2,3,1)	(2,4,1)
(1,3,4,2,5):	(4,3,1)	(4,5,2)	(3,2)(5,4)	(5,2,1)	(4,3,2)	(2,3,5,4,1)	(2,1)(5,4)
(5,4,3,2,1):	(2,3,4,5,1)	(4,1)(5,3)	(4,1)(5,2)	(4,2)(5,3)	(4,5,1)	(3,1)(5,2)	(3,5,2,4,1)
(5,2,1,4,3):	(5,1)(3,2)	(2,5,3,4,1)	(2,4,1)	(4,5,3)	(2,5,1)	(3,1)(4,2)	(3,4,1)
(5,4,1,3,2):	(2,1)(4,3)	(4,5,1)	(4,5,2,3,1)	(5,1)(4,2)	(4,3,1)	(3,5,2)	(4,5,2)
(5,3,2,4,1):	(2,1)(5,4)	(3,5,1)	(3,1)(5,2)	(4,2,3,5,1)	(3,1)(5,4)	(5,2)(4,3)	(3,5,2)
(5,3,1,2,4):	(5,4,2)	(3,2,1)	(3,5,1)	(4,1)(3,2)	(3,5,4,2,1)	(5,1)(4,3)	(5,1)(3,2)
(5,2,4,3,1):	(3,2)(5,4)	(2,1)(5,3)	(2,5,1)	(4,1)(5,3)	(2,1)(5,4)	(3,4,2,5,1)	(3,5,1)

	(5,4,1,3,2):	(5,1,2,4,3):	(5,2,1,3,4):	(5,1,4,3,2):	(5,4,1,2,3):
(1,2,3,4,5):	(4,2,1)	(3,1)(5,2)	(4,1)(5,3)	(5,2,1)	(3,1)(4,2)
(1,2,5,3,4):	(4,2,3,5,1)	(5,2,1)	(4,3,1)	(3,5,2)	(5,1)(4,2)
(1,4,5,2,3):	F	(3,2,5,4,1)	(4,2)(5,3)	(5,4,1)	(3,2,1)
(1,4,2,3,5):	(5,2,1)	(3,1)(5,4)	(5,3,2,4,1)	(2,1)(5,4)	(3,1)(5,2)
(1,3,5,4,2):	(5,2,4,3,1)	(4,5,3)	(5,1)(3,2)	(2,4,5,3,1)	(5,2)(4,3)
(1,3,4,2,5):	(4,3,5,2,1)	(5,3,1)	(4,1)(3,2)	(2,1)(5,3)	(5,2,4,3,1)
(5,4,3,2,1):	(2,4,5,3,1)	(3,4,2)	(2,5,1)	(3,1)(4,2)	(4,5,3)
(5,2,1,4,3):	(5,3,1)	(3,2)(5,4)	(5,1)(4,2)	(3,1)(5,4)	(5,3,2)
(5,4,1,3,2):	(2,4,3,5,1)	(5,3,4,2,1)	(2,3,1)	(4,2)(5,3)	(5,1)(4,3)
(5,3,2,4,1):	(2,3,4,5,1)	(4,2,1)	(2,5,4,3,1)	(3,4,2)	(4,5,1)
(5,3,1,2,4):	(3,4,2)	(4,1)(5,2)	(5,4,3)	(5,2,3,4,1)	(4,2,1)
(5,2,4,3,1):	(4,5,3)	(4,1)(3,2)	(5,4,2)	(3,4,1)	(4,5,3,2,1)

GROUP (1,2,3,4,5)
K2*K4

	(1,2,3):	(2,3,4):	(1,3,4):	(1,2,4):	(1,2,5):	(1,3,5):	(2,3,5):
(1,2,3,4,5):	(3,2,4,5,1)	(2,4,3,5,1)	(4,2,3,5,1)	(3,4,2,5,1)	(3,4,5,2,1)	(4,5,2,3,1)	(2,4,5,3,1)
(1,2,5,3,4):	(5,3,2,4,1)	(2,4,5,3,1)	(4,2,5,3,1)	(5,3,4,2,1)	(5,2,3,4,1)	(4,1)(5,2)	(2,4,1)
(1,4,5,2,3):	(3,4,5,2,1)	(4,3,5,2,1)	(3,5,2)	(3,1)(5,4)	(3,1)(5,4)	(3,2)(5,4)	(4,5,3,2,1)
(1,4,2,3,5):	(3,4,2,5,1)	(4,3,2,5,1)	(5,1)(3,2)	(3,5,1)	(3,5,4,2,1)	(5,4,2,3,1)	(4,2,5,3,1)
(1,3,5,4,2):	(5,4,2)	(3,2,5,4,1)	(5,4,3,2,1)	(5,4,3)	(4,2)(5,3)	(5,3,4,2,1)	(3,4,2,5,1)
(1,3,4,2,5):	(5,1)(4,2)	(3,2,4,5,1)	(4,3,2,5,1)	(5,1)(4,3)	(5,3,4,2,1)	(4,2,5,3,1)	(3,1)(4,2)
(5,4,3,2,1):	(5,4,3)	(5,4,1)	(2,1)(5,4)	(3,2)(5,4)	(4,3,2)	(2,1)(4,3)	(5,1)(4,3)
(5,2,1,4,3):	(5,2)(4,3)	(4,1)(5,2)	(5,2,1)	(3,5,2)	(5,4,3)	(5,4,3,2,1)	(4,3,2,5,1)
(5,4,1,3,2):	(5,4,1)	(3,1)(5,4)	(2,5,4,3,1)	(5,4,3,2,1)	(5,3,2,4,1)	(2,5,3,4,1)	(3,4,1)
(5,3,2,4,1):	(4,1)(5,3)	(5,3,1)	(2,4,5,3,1)	(4,5,3,2,1)	(4,1)(3,2)	(2,4,1)	(5,4,1)
(5,3,1,2,4):	(4,5,3,2,1)	(2,1)(5,3)	(4,2)(5,3)	(4,2,5,3,1)	(4,5,2,3,1)	(4,5,2)	(2,1)(5,4)
(5,2,4,3,1):	(4,3,5,2,1)	(5,2,1)	(4,5,2)	(4,5,2,3,1)	(4,3,1)	(4,3,2)	(5,4,3,2,1)

	(1,4,5):	(2,4,5):	(3,4,5):	(3,2,1):	(4,3,2):	(4,3,1):	(4,2,1):
(1,2,3,4,5):	(5,2,3,4,1)	(2,5,3,4,1)	(2,3,5,4,1)	(4,5,1)	(2,5,1)	(5,1)(3,2)	(5,1)(4,3)
(1,2,5,3,4):	(5,2)(4,3)	(2,1)(4,3)	(2,5,4,3,1)	(4,1)(5,3)	(2,1)(5,3)	(5,3,2)	(4,5,3)
(1,4,5,2,3):	(5,4,2,3,1)	(4,2,5,3,1)	(4,2,3,5,1)	(4,5,2)	(4,1)(5,2)	(5,2,3,4,1)	(5,2,4,3,1)
(1,4,2,3,5):	(2,3,5,4,1)	(4,1)(5,3)	(4,1)(3,2)	(5,1)(4,2)	(4,5,1)	(2,3,4,5,1)	(2,4,3,5,1)
(1,3,5,4,2):	(2,1)(5,3)	(3,5,1)	(3,2,1)	(5,4,2,3,1)	(3,1)(5,4)	(2,1)(5,4)	(2,3,5,4,1)
(1,3,4,2,5):	(2,5,3,4,1)	(3,4,1)	(3,2,5,4,1)	(4,2,3,5,1)	(3,5,1)	(2,5,1)	(2,3,4,5,1)
(5,4,3,2,1):	(3,2,1)	(5,1)(3,2)	(5,2,1)	(2,5,4,3,1)	(5,4,2,3,1)	(3,5,4,2,1)	(3,2,5,4,1)
(5,2,1,4,3):	(3,5,4,2,1)	(4,2,3,5,1)	(4,2,1)	(5,2,4,3,1)	(4,5,2,3,1)	(3,4,5,2,1)	(3,5,2,4,1)
(5,4,1,3,2):	(5,3,2)	(3,2,1)	(3,1)(5,2)	(2,3,6,4,1)	(3,5,4,2,1)	(5,4,2)	(3,2)(5,4)
(5,3,2,4,1):	(4,3,2)	(5,4,3,2,1)	(5,2,4,3,1)	(2,5,3,4,1)	(5,3,4,2,1)	(4,2)(5,3)	(5,3,2)
(5,3,1,2,4):	(5,2,4,3,1)	(2,5,4,3,1)	(2,4,3,5,1)	(4,5,3)	(2,5,3,4,1)	(5,3,2,4,1)	(5,3,1)
(5,2,4,3,1):	(3,1)(4,2)	(5,4,2,3,1)	(5,1)(4,2)	(5,4,2)	(5,4,3)	(3,5,2,4,1)	(3,1)(5,2)

	(5,2,1):	(5,3,1):	(5,3,2):	(5,4,1):	(5,4,2):	(5,4,3):
(1,2,3,4,5):	(4,5,3)	(3,2)(5,4)	(2,1)(5,4)	(3,4,2)	(2,1)(4,3)	(2,3,1)
(1,2,5,3,4):	(3,4,1)	(3,2,5,4,1)	(2,3,5,4,1)	(3,4,2,5,1)	(2,3,4,5,1)	(2,5,1)
(1,4,5,2,3):	(2,4,5,3,1)	(2,3,4,5,1)	(4,5,1)	(2,3,1)	(4,3,1)	(4,1)(3,2)
(1,4,2,3,5):	(4,2)(5,3)	(3,4,2)	(4,2,1)	(3,5,2)	(4,3,5,2,1)	(4,5,2,3,1)
(1,3,5,4,2):	(4,2,3,5,1)	(4,2,1)	(3,1)(4,2)	(4,3,5,2,1)	(3,5,2,4,1)	(3,4,5,2,1)
(1,3,4,2,5):	(5,4,2)	(5,4,2)	(3,5,4,2,1)	(5,2)(4,3)	(3,4,5,2,1)	(3,1)(5,2)
(5,4,3,2,1):	(4,3,2,5,1)	(4,3,2,5,1)	(5,2,4,3,1)	(4,5,3,2,1)	(3,4,2,5,1)	(5,3,4,2,1)
(5,2,1,4,3):	(2,4,3,5,1)	(2,1)(4,3)	(4,3,1)	(2,1)(5,3)	(4,1)(5,3)	(4,5,3,2,1)
(5,4,1,3,2):	(4,1)(3,2)	(4,1)(5,2)	(3,5,2,4,1)	(5,2,3,4,1)	(3,2,4,5,1)	(3,4,2,5,1)
(5,3,2,4,1):	(3,2,5,4,1)	(3,5,2,4,1)	(5,2,3,4,1)	(3,1)(4,2)	(5,1)(3,2)	(5,1)(4,2)
(5,3,1,2,4):	(3,1)(5,4)	(3,2,4,5,1)	(2,3,4,5,1)	(3,4,5,3,1)	(2,3,1)	(2,4,1)
(5,2,4,3,1):	(2,5,4,3,1)	(2,4,3,5,1)	(5,1)(4,3)	(2,4,5,3,1)	(5,3,1)	(5,3,2,4,1)

GROUP (1,2,3,4,5)
K2*K5

	(1,2)(3,4):	(1,3)(2,4):	(1,4)(2,3):	(1,2)(3,5):	(1,3)(2,5):	(1,5)(2,3):	(1,2)(4,5):
(1,2,3,4,5):	(3,5,1)	(4,3,2,5,1)	(5,1)(4,2)	(3,1)(5,4)	(4,5,3,2,1)	(4,5,2)	(3,4,1)
(1,2,5,3,4):	(5,3,1)	(4,5,3,2,1)	(4,2)(5,3)	(5,4,1)	(4,1)(3,2)	(3,5,2,4,1)	(5,1)(4,3)
(1,4,5,2,3):	(3,5,2,4,1)	(5,2)(4,3)	(5,2,1)	(3,2,4,5,1)	(4,5,3)	(2,1)(5,4)	(3,1)(4,2)
(1,4,2,3,5):	(3,2,4,5,1)	(5,1)(4,3)	(2,5,1)	(3,1)(4,2)	(5,3,4,2,1)	(5,4,2)	(3,5,2,4,1)
(1,3,5,4,2):	(3,2)(5,4)	(5,4,1)	(2,5,4,3,1)	(3,4,2)	(5,1)(4,2)	(4,2,5,3,1)	(3,5,2)
(1,3,4,2,5):	(5,1)(3,2)	(4,5,1)	(2,4,3,5,1)	(5,4,2,3,1)	(4,2,1)	(4,2)(5,3)	(5,2,3,4,1)
(5,4,3,2,1):	(5,4,2)	(2,3,5,4,1)	(3,1)(5,4)	(5,2)(4,3)	(2,4,3,5,1)	(4,3,1)	(5,3,2)
(5,2,1,4,3):	(4,5,2)	(5,2,3,4,1)	(3,1)(5,2)	(4,3,2)	(5,1)(4,3)	(2,5,4,3,1)	(4,2)(5,3)
(5,4,1,3,2):	(5,4,2,3,1)	(2,1)(5,4)	(5,4,3)	(5,2,3,4,1)	(2,4,1)	(4,1)(5,3)	(5,1)(3,2)
(5,3,2,4,1):	(4,2,5,3,1)	(2,1)(5,3)	(4,5,3)	(4,1)(5,2)	(2,3,5,4,1)	(3,4,1)	(4,3,2,5,1)
(5,3,1,2,4):	(4,1)(5,3)	(5,3,2)	(5,3,4,2,1)	(4,5,1)	(3,2)(5,4)	(3,4,5,2,1)	(4,3,1)
(5,2,4,3,1):	(4,1)(5,2)	(3,5,2)	(3,4,5,2,1)	(4,3,2,5,1)	(5,4,3)	(2,1)(4,3)	(4,2,5,3,1)

	(1,4)(2,5):	(1,5)(2,4):	(1,3)(4,5):	(1,4)(3,5):	(1,5)(3,4):	(2,3)(4,5):	(2,4)(3,5):
(1,2,3,4,5):	(5,3,4,2,1)	(5,2)(4,3)	(4,1)(3,2)	(5,4,2,3,1)	(3,5,2)	(2,4,1)	(2,5,4,3,1)
(1,2,5,3,4):	(3,4,2)	(3,4,5,2,1)	(4,3,2,5,1)	(5,4,2)	(3,1)(5,2)	(2,4,3,5,1)	(2,1)(5,4)
(1,4,5,2,3):	(5,3,1)	(2,5,4,3,1)	(3,4,2)	(5,1)(3,2)	(2,3,5,4,1)	(4,2,1)	(4,3,2,5,1)
(1,4,2,3,5):	(2,1)(5,3)	(5,4,3)	(5,2,3,4,1)	(2,3,1)	(3,2)(5,4)	(4,1)(5,2)	(4,3,1)
(1,3,5,4,2):	(2,4,3,5,1)	(4,1)(5,3)	(5,2,1)	(2,1)(4,3)	(4,5,3,2,1)	(3,1)(5,2)	(3,4,1)
(1,3,4,2,5):	(2,1)(4,3)	(4,5,3)	(4,1)(5,2)	(2,5,4,3,1)	(5,3,2)	(3,5,2,4,1)	(3,1)(5,4)
(5,4,3,2,1):	(3,2,4,5,1)	(4,1)(3,2)	(2,1)(5,3)	(3,4,5,2,1)	(4,2,1)	(5,1)(4,3)	(5,2,3,4,1)
(5,2,1,4,3):	(3,5,1)	(2,3,5,4,1)	(5,3,4,2,1)	(3,2,1)	(2,1)(5,4)	(4,2,5,3,1)	(4,1)(3,2)
(5,4,1,3,2):	(4,3,2)	(4,5,3,2,1)	(2,5,1)	(5,2)(4,3)	(4,2,5,3,1)	(3,5,1)	(3,4,5,2,1)
(5,3,2,4,1):	(3,2)(5,4)	(3,2,1)	(2,4,3,5,1)	(4,5,2)	(3,1)(4,2)	(5,1)(4,3)	(5,2,1)
(5,3,1,2,4):	(5,4,2,3,1)	(3,1)(5,2)	(4,3,2)	(5,1)(4,2)	(3,5,2,4,1)	(2,1)(4,3)	(2,5,1)
(5,2,4,3,1):	(3,1)(5,4)	(2,3,1)	(4,2)(5,3)	(3,4,5,2,1)	(2,4,1)	(3,1)(5,4)	(4,3,1)

	(2,5)(3,4):
(1,2,3,4,5):	(2,1)(5,3)
(1,2,5,3,4):	(2,3,1)
(1,4,5,2,3):	(4,1)(5,3)
(1,4,2,3,5):	(4,5,3,2,1)
(1,3,5,4,2):	(3,2,4,5,1)
(1,3,4,2,5):	(3,2,1)
(5,4,3,2,1):	(5,1)(4,2)
(5,2,1,4,3):	(4,5,1)
(5,4,1,3,2):	(3,1)(4,2)
(5,3,2,4,1):	(5,4,3,1)
(5,3,1,2,4):	(2,3,5,4,1)
(5,2,4,3,1):	(5,4,1)

GROUP (1,2,3,4,5)
K3*K2

	(1,2,3,4,5):	(1,2,5,3,4):	(1,4,5,2,3):	(1,4,2,3,5):	(1,3,5,4,2):	(1,3,4,2,5):	(5,4,3,2,1):
(1,3,5,2,4):	(4,2,5,3,1)	(4,3,1)	(5,4,2)	(5,3,2)	(5,1)(3,2)	(5,3,1)	(2,3,4,5,1)
(1,5,4,2,3):	(3,2,1)	(3,2,4,5,1)	(2,1)(5,3)	(2,1)(4,3)	(5,2)(4,3)	(4,3,2)	(4,1)(5,2)
(1,5,3,4,2):	(4,3,2)	(3,2)(5,4)	(2,4,3,5,1)	(2,4,1)	(4,1)(5,2)	(4,1)(3,2)	(3,1)(5,2)
(1,2,5,4,3):	(5,2,1)	(5,1)(4,2)	(3,2,1)	(3,4,5,2,1)	(4,5,3)	(4,5,2)	(4,1)(5,3)
(1,5,2,3,4):	(3,1)(4,2)	(3,1)(5,4)	(4,2)(5,3)	(4,3,2)	(4,3,2,5,1)	(4,3,1)	(2,5,1)
(1,4,5,3,2):	(5,4,3)	(3,5,2)	(5,1)(4,3)	(5,4,1)	(2,4,1)	(2,3,5,4,1)	(3,1)(4,2)
(5,3,1,4,2):	(5,4,3,2,1)	(5,1)(3,2)	(2,1)(4,3)	(2,1)(5,4)	(4,5,2)	(3,2)(5,4)	(3,5,2,4,1)
(5,4,1,3,2):	(5,3,1)	(5,2,4,3,1)	E	(4,5,3)	(2,3,4,5,1)	(5,2,1)	(4,2,3,5,1)
(5,1,2,4,3):	(4,1)(5,2)	(4,2,1)	(3,2,5,4,1)	(3,1)(5,2)	(5,3,1)	(5,2,1)	(4,5,3)
(5,2,1,3,4):	(4,2)(5,3)	(5,4,3)	(5,1)(4,2)	(5,3,2,4,1)	(4,1)(3,2)	(4,1)(5,3)	(2,3,1)
(5,1,4,3,2):	(5,4,1)	(5,2,1)	(3,4,1)	(3,1)(5,4)	(2,4,5,3,1)	(2,1)(5,4)	(4,2)(5,3)
(5,4,1,2,3):	(3,1)(5,2)	(3,1)(4,2)	(5,3,2)	(5,2)(4,3)	(5,1)(4,3)	(5,2,4,3,1)	(4,5,1)

	(5,2,1,4,3):	(5,4,1,3,2):	(5,3,2,4,1):	(5,3,1,2,4):	(5,2,4,3,1):
(1,3,5,2,4):	(3,2)(5,4)	(5,1)(4,3)	(2,1)(4,3)	(4,2,1)	(2,1)(5,4)
(1,5,4,2,3):	(2,5,3,4,1)	(4,5,2)	(4,5,1)	(3,5,1)	(4,1)(5,3)
(1,5,3,4,2):	(2,5,1)	(4,5,2,3,1)	(3,1)(5,4)	(5,4,3)	(3,5,1)
(1,2,5,4,3):	(3,4,1)	(4,2)(5,3)	(4,2,3,5,1)	(5,1)(3,2)	(4,1)(3,2)
(1,5,2,3,4):	(5,3,2)	(4,5,2,3,1)	(2,1)(5,4)	(3,5,4,2,1)	(2,1)(5,3)
(1,5,2,3,4):	(5,3,2)	(4,5,2,3,1)	(2,1)(5,4)	(3,5,4,2,1)	(2,1)(5,3)
(1,4,5,3,2):	(5,2,1)	(4,3,1)	(3,4,2,5,1)	(5,2)(4,3)	(3,4,2,5,1)
(1,4,2,3,5):	(2,1)(5,3)	(3,4,5,2,1)	(3,4,2)	(5,4,3)	(3,4,1)
(1,3,5,4,2):	(3,2,4,5,1)	(5,2,3,4,1)	(5,2,1)	(4,3,1)	(4,2,1)
(1,3,4,2,5):	(3,2,1)	(5,4,3)	(4,1)(5,2)	(4,1)(5,3)	(3,2)(5,4)
(5,4,3,2,1):	(5,1)(4,2)	(4,5,3)	(4,3,2,5,1)	(5,2,1)	(4,5,3,2,1)
(5,2,1,4,3):	(4,5,1)	(3,5,2)	(3,4,5,2,1)	(4,1)(3,2)	(3,4,1)
(5,4,1,3,2):	(3,1)(4,2)	(5,2,1)	(2,5,4,3,1)	(4,3,2)	(2,5,1)
(5,3,2,4,1):	(5,4,3,1)	(2,1)(5,3)	(3,5,2)	(5,2,3,4,1)	(4,1)(5,2)
(5,3,1,2,4):	(4,5,3)	(5,1)(4,2)	(4,2,1)	(4,1)(5,2)	(4,5,3,2,1)
(5,2,4,3,1):	(4,5,3)	(5,1)(4,2)	(4,2,1)	(3,2,1)	(4,5,3,2,1)

GROUP (1,2,3,4,5)
K3*K4

	(1,2,3):	(2,3,4):	(1,3,4):	(1,2,4):	(1,2,5):	(1,3,5):	(2,3,5):
(1,3,5,2,4):	(4,1)(5,2)	(3,1)(5,2)	(5,2,4,3,1)	(4,3,5,2,1)	(4,1)(5,3)	(5,3,2,4,1)	(3,2,5,4,1)
(1,5,4,2,3):	(3,5,4,2,1)	(5,4,3,2,1)	(3,2)(5,4)	(3,1)(5,4)	(3,1)(4,2)	(3,4,2)	(5,3,4,2,1)
(1,5,3,4,2):	(4,2)(5,3)	(5,3,2,4,1)	(4,5,3,2,1)	(4,5,3)	(3,4,2)	(4,2,1)	(5,1)(4,2)
(1,2,5,4,3):	(5,4,3,2,1)	(2,1)(5,4)	(5,4,2)	(5,4,2,3,1)	(5,2,4,3,1)	(5,2)(4,3)	(2,1)(4,3)
(1,5,2,3,4):	(3,5,2,4,1)	(5,2,4,3,1)	(4,5,2,3,1)	(3,4,5,2,1)	(3,4,1)	(4,1)(3,2)	(5,3,2,4,1)
(1,4,5,3,2):	(4,5,3)	(4,1)(5,3)	(2,1)(5,3)	(5,3,2)	(3,2)(5,4)	(2,1)(5,4)	(4,5,1)
(5,3,1,4,2):	(5,3,4,2,1)	(4,5,3,2,1)	(5,3,2)	(5,3,1)	(5,4,2,3,1)	(5,4,2)	(4,2,1)
(5,4,1,3,2):	(5,4,1)	(3,1)(5,4)	(2,5,4,3,1)	(5,4,3,2,1)	(5,3,2,4,1)	(2,5,3,4,1)	(3,4,1)
(5,1,2,4,3):	(4,3,2,5,1)	(2,5,1)	(5,1)(4,2)	(4,2,3,5,1)	(4,3,5,2,1)	(5,2,4,3,1)	(2,5,4,3,1)
(5,2,1,3,4):	(4,5,2)	(3,5,2,4,1)	(4,3,5,2,1)	(5,2)(4,3)	(4,5,3)	(4,5,3,2,1)	(3,2,4,5,1)
(5,1,4,3,2):	(5,1)(4,3)	(4,5,1)	(2,5,1)	(5,1)(3,2)	(3,5,4,2,1)	(2,5,4,3,1)	(4,3,1)
(5,4,1,2,3):	(3,2,5,4,1)	(2,5,4,3,1)	(5,4,2,3,1)	(3,5,4,2,1)	(3,5,2,4,1)	(5,2,3,4,1)	(2,5,3,4,1)

	(1,4,5):	(2,4,5):	(3,4,5):	(3,2,1):	(4,3,2):	(4,3,1):	(4,2,1):
(1,3,5,2,4):	(4,2)(5,3)	(3,5,4,2,1)	(3,1)(4,2)	(5,2,3,4,1)	(3,4,5,2,1)	(4,5,2)	(3,5,2)
(1,5,4,2,3):	(2,3,1)	(5,3,1)	(5,1)(3,2)	(5,4,2)	(5,4,1)	(2,3,5,4,1)	(2,5,4,3,1)
(1,5,3,4,2):	(2,1)(4,3)	(5,1)(4,3)	(5,4,3,2,1)	(4,2,5,3,1)	(5,3,1)	(2,1)(5,3)	(2,5,3,4,1)
(1,2,5,4,3):	(3,1)(5,2)	(2,3,1)	(2,5,1)	(5,4,3)	(2,3,5,4,1)	(3,2,5,4,1)	(3,1)(5,4)
(1,5,2,3,4):	(3,4,2)	(5,3,4,2,1)	(5,4,2,3,1)	(4,1)(5,2)	(5,2,1)	(3,5,2)	(5,2)(4,3)
(1,4,5,3,2):	(5,4,3,2,1)	(4,3,2,5,1)	(4,3,5,2,1)	(2,4,5,3,1)	(4,2,5,3,1)	(5,3,4,2,1)	(5,3,2,4,1)
(5,3,1,4,2):	(2,5,4,3,1)	(4,3,1)	(4,3,2,5,1)	(4,2)(5,3)	(4,1)(5,3)	(2,5,3,4,1)	(2,4,5,3,1)
(5,4,1,3,2):	(5,3,2)	(3,2,1)	(3,1)(5,2)	(2,3,5,4,1)	(3,5,4,2,1)	(5,4,2)	(3,2)(5,4)
(5,1,2,4,3):	(3,5,2,4,1)	(2,3,5,4,1)	(2,4,1)	(5,1)(4,3)	(2,3,4,5,1)	(3,2,4,5,1)	(3,5,1)
(5,2,1,3,4):	(5,3,4,2,1)	(3,4,2,5,1)	(3,5,4,2,1)	(4,5,2,3,1)	(3,1)(5,2)	(5,2,4,5,1)	(5,2,3,4,1)
(5,1,4,3,2):	(3,2,5,4,1)	(4,1)(3,2)	(4,1)(5,2)	(2,4,3,5,1)	(4,2,3,5,1)	(3,4,2,5,1)	(3,2,4,5,1)
(5,4,1,2,3):	(3,5,2)	(2,1)(5,3)	(2,3,1)	(5,4,1)	(2,1)(5,4)	(3,2)(5,4)	(5,4,3)

	(5,2,1):	(5,3,1):	(5,3,2):	(5,4,1):	(5,4,2):	(5,4,3):
(1,3,5,2,4):	(2,3,5,4,1)	(2,4,1)	(3,4,1)	(2,4,3,5,1)	(3,5,1)	(3,2,4,5,1)
(1,5,4,2,3):	(4,2,5,3,1)	(4,2,4,5,1)	(5,1)(4,2)	(4,5,2,3,1)	(5,2,4,3,1)	(5,2,3,4,1)
(1,5,3,4,2):	(3,4,2,5,1)	(3,5,4,2,1)	(5,4,2,3,1)	(3,4,5,2,1)	(5,2,3,4,1)	(5,2,1)
(1,2,5,4,3):	(4,3,1)	(4,3,2,5,1)	(2,4,3,5,1)	(4,2,5,3,1)	(2,4,5,3,1)	(2,5,3,4,1)
(1,5,2,3,4):	(2,5,3,4,1)	(2,3,5,4,1)	(5,4,1)	(2,3,4,5,1)	(5,1)(4,3)	(5,1)(3,2)
(1,4,5,3,2):	(3,2,4,5,1)	(3,4,5,2,1)	(4,5,2,3,1)	(3,2,1)	(4,1)(3,2)	(4,2,1)
(5,3,1,4,2):	(3,1)(4,2)	(3,4,2,5,1)	(4,2,3,5,1)	(3,1)(5,2)	(4,5,2,3,1)	(4,1)(5,2)
(5,4,1,3,2):	(4,1)(3,2)	(4,1)(5,2)	(3,5,2,4,1)	(4,3,2,5,1)	(3,2,4,5,1)	(3,4,2,5,1)
(5,1,2,4,3):	(5,4,3)	(4,3,2)	(2,1)(4,3)	(4,2)(5,3)	(2,1)(5,3)	(2,4,5,3,1)
(5,2,1,3,4):	(2,3,4,5,1)	(2,1)(5,4)	(3,1)(5,4)	(2,1)(4,3)	(3,4,1)	(3,2,1)
(5,1,4,3,2):	(4,3,2)	(5,2)(4,3)	(4,3,5,2,1)	(5,3,2)	(4,5,3,2,1)	(4,2,5,3,1)
(5,4,1,2,3):	(4,1)(5,3)	(4,1)(3,2)	(2,4,1)	(4,2,3,5,1)	(2,4,1,3,5)	(2,3,4,5,1)

GROUP (1,2,3,4,5)
K3*K5

	(1,2)(3,4):	(1,3)(2,4):	(1,4)(2,3):	(1,2)(3,5):	(1,3)(2,5):	(1,5)(2,3):	(1,2)(4,5):
(1,3,5,2,4):	(4,5,2,3,1)	(5,2,1)	(5,2)(4,3)	(4,1)(3,2)	(5,4,1)	(2,5,3,4,1)	(4,2,3,5,1)
(1,5,4,2,3):	(3,2,5,4,1)	(5,4,3)	(2,1)(5,4)	(3,4,2,5,1)	(4,2)(5,3)	(4,2,1)	(3,1)(5,2)
(1,5,3,4,2):	(5,3,2)	(4,1)(5,3)	(2,4,5,3,1)	(5,4,2)	(4,2,3,5,1)	(3,1)(4,2)	(5,2)(4,3)
(1,2,5,4,3):	(5,4,1)	(3,2)(5,4)	(3,5,4,2,1)	(5,1)(4,3)	(4,3,2)	(4,3,5,2,1)	(5,3,1)
(1,5,2,3,4):	(3,1)(5,2)	(4,3,5,2,1)	(4,5,2)	(3,2,5,4,1)	(4,1)(5,3)	(2,4,1)	(3,4,2,5,1)
(1,4,5,3,2):	(4,2)(5,3)	(2,5,3,4,1)	(5,3,1)	(4,5,2)	(2,3,4,5,1)	(3,1)(5,4)	(4,3,2)
(5,3,1,4,2):	(5,3,2,4,1)	(4,5,3)	(2,1)(5,3)	(5,1)(4,2)	(3,4,2)	(3,5,4,2,1)	(5,2,4,3,1)
(5,4,1,3,2):	(5,4,2,3,1)	(2,1)(5,4)	(5,4,3)	(5,2,3,4,1)	(2,4,1)	(4,1)(5,3)	(5,1)(3,2)
(5,1,2,4,3):	(4,5,1)	(5,1)(3,2)	(3,4,2,5,1)	(4,3,1)	(5,4,3,2,1)	(5,2)(4,3)	(4,1)(5,3)
(5,2,1,3,4):	(3,5,2)	(4,1)(5,2)	(5,2,4,3,1)	(3,2)(5,4)	(4,5,1)	(2,4,5,3,1)	(3,4,2)
(5,1,4,3,2):	(5,1)(4,2)	(2,3,4,5,1)	(3,5,1)	(5,2,4,3,1)	(2,1)(4,3)	(5,4,3)	(5,3,2,4,1)
(5,4,1,2,3):	(3,1)(5,4)	(5,4,3,2,1)	(5,4,2)	(3,4,1)	(5,3,2,4,1)	(4,1)(5,2)	(3,5,1)

	(1,4)(2,5):	(1,5)(2,4):	(1,3)(4,5):	(1,4)(3,5):	(1,5)(3,4):	(2,3)(4,5):	(2,4)(3,5):
(1,3,5,2,4):	(5,4,3)	(2,1)(5,3)	(5,1)(4,2)	(4,3,2)	(4,1)(3,2)	(3,4,2,5,1)	(3,2,1)
(1,5,4,2,3):	(2,4,5,3,1)	(4,3,1)	(3,5,2)	(2,3,4,5,1)	(4,1)(3,2)	(5,2,1)	(5,1)(4,3)
(1,5,3,4,2):	(2,3,4,5,1)	(3,4,1)	(4,3,5,2,1)	(2,1)(5,4)	(3,4,2,1)	(5,2,4,3,1)	(5,4,1)
(1,2,5,4,3):	(3,1)(4,2)	(4,5,2,3,1)	(5,3,2)	(3,4,2,5,1)	(3,2,1)	(2,1)(5,3)	(2,3,4,5,1)
(1,5,2,3,4):	(4,5,3)	(2,1)(4,3)	(4,2,3,5,1)	(3,2)(5,4)	(2,3,1)	(5,1)(4,3)	(5,2)(4,3)
(1,4,5,3,2):	(5,1)(3,2)	(3,2,5,4,1)	(2,1)(4,3)	(5,2,1)	(2,4,1)	(4,1)(5,3)	(4,3,2)
(5,3,1,4,2):	(2,3,1)	(3,1)(5,4)	(5,2)(4,3)	(2,5,1)	(5,2,4,3,1)	(2,4,5,3,1)	(5,3,2,4,1)
(5,4,1,3,2):	(4,3,2)	(4,3,2,5,1)	(3,5,2,4,1)	(5,2)(4,3)	(4,5,1)	(3,5,4,2,1)	(4,2,1)
(5,1,2,4,3):	(3,5,2,1)	(4,1)(5,2)	(5,2,4,3,1)	(3,2)(5,4)	(2,1)(4,3)	(5,4,3)	(5,3,2,4,1)
(5,2,1,3,4):	(5,1)(4,3)	(2,3,4,5,1)	(4,2,1)	(5,2,4,3,1)	(5,3,2,4,1)	(4,1)(5,2)	(3,5,1)
(5,1,4,3,2):	(4,2,1)	(5,4,3,2,1)	(2,5,3,4,1)	(3,4,1)			
(5,4,1,2,3):	(2,4,5,3,1)						

	(2,5)(3,4):
(1,3,5,2,4):	(3,1)(5,4)
(1,5,4,2,3):	(5,3,2,4,1)
(1,5,3,4,2):	(5,1)(3,2)
(1,2,5,4,3):	(2,4,1)
(1,5,2,3,4):	(5,3,1)
(1,4,5,3,2):	(4,2,3,5,1)
(5,3,1,4,2):	(4,1)(3,2)
(5,4,1,3,2):	(3,1)(4,2)
(5,1,2,4,3):	(2,1)(5,4)
(5,2,1,3,4):	(3,5,1)
(5,1,4,3,2):	(4,2,1)
(5,4,1,2,3):	(2,4,5,3,1)

GROUP (1,2,3,4,5)
K4*K2

	(1,2,3,4,5):	(1,2,5,3,4):	(1,4,5,2,3):	(1,4,2,3,5):	(1,3,5,4,2):	(1,3,4,2,5):	(5,4,3,2,1):
(1,2,3):	(3,4,5,2,1)	(3,4,2,5,1)	(4,5,3,2,1)	(4,3,5,2,1)	(5,4,3)	(5,2)(4,3)	(5,4,1)
(2,3,4):	(3,2,4,5,1)	(3,2,5,4,1)	(2,4,5,3,1)	(2,4,3,5,1)	(4,3,5,2,1)	(4,3,2,5,1)	(5,2,1)
(1,3,4):	(2,4,5,3,1)	(2,5,4,3,1)	(4,5,2)	(4,2)(5,3)	(4,2,3,5,1)	(4,2,5,3,1)	(5,1)(3,2)
(1,2,4):	(4,5,2,3,1)	(4,2,5,3,1)	(3,2)(5,4)	(3,5,2)	(3,5,1)	(3,1)(5,2)	(5,1)(4,3)
(1,2,5):	(5,2,3,4,1)	(5,3,4,2,1)	(4,1)(3,2)	(4,5,2,3,1)	(3,1)(5,4)	(3,4,5,2,1)	(5,4,3)
(1,3,5):	(2,5,3,4,1)	(2,1)(4,3)	(4,1)(5,2)	(4,2,5,3,1)	(5,4,2,3,1)	(5,3,4,2,1)	(3,2)(5,4)
(2,3,5):	(3,4,2,5,1)	(3,4,1)	(4,2,5,3,1)	(4,3,2,5,1)	(5,4,3,2,1)	(5,1)(4,3)	(2,1)(5,4)
(1,4,5):	(2,3,5,4,1)	(2,1)(5,3)	(5,2,3,4,1)	(5,4,2,3,1)	(3,1)(4,2)	(3,5,4,2,1)	(4,3,2)
(2,4,5):	(4,2,3,5,1)	(4,1)(5,3)	(5,4,2,3,1)	(5,1)(3,2)	(3,2,1)	(3,5,1)	(2,1)(4,3)
(3,4,5):	(2,4,3,5,1)	(2,3,5,4,1)	(5,2,4,3,1)	(5,1)(4,2)	(4,2,1)	(4,2,3,5,1)	(3,2,1)
(3,2,1):	(4,5,3)	(5,2)(4,3)	(4,5,1)	(4,1)(5,3)	(2,3,5,4,1)	(2,5,3,4,1)	(5,4,2,3,1)
(4,3,2):	(4,5,1)	(4,1)(5,2)	(3,1)(5,4)	(3,5,1)	(2,1)(5,3)	(2,5,1)	(5,3,4,2,1)
(4,3,1):	(2,1)(5,4)	(2,5,1)	(3,4,5,2,1)	(3,5,4,2,1)	(4,2)(5,3)	(5,4,2)	(5,3,2,4,1)
(4,2,1):	(3,2)(5,4)	(5,3,2)	(2,3,4,5,1)	(2,3,5,4,1)	(3,5,2,4,1)	(3,2,5,4,1)	(5,2,4,3,1)
(5,2,1):	(3,4,2)	(4,5,3)	(4,2,3,5,1)	(4,1)(3,2)	(3,2,5,4,1)	(3,4,1)	(2,5,4,3,1)
(5,3,1):	(2,1)(4,3)	(2,3,4,5,1)	(4,3,5,2,1)	(4,2,1)	(5,4,2)	(3,4,2)	(3,2,5,4,1)
(5,3,2):	(5,1)(4,3)	(5,2,3,4,1)	(4,3,1)	(4,5,1)	(2,1)(5,4)	(2,3,4,5,1)	(3,5,4,2,1)
(5,4,1):	(2,3,1)	(2,4,5,3,1)	(3,5,2)	(3,4,2)	(3,4,2,5,1)	(3,1)(4,2)	(4,3,2,5,1)
(5,4,2):	(5,1)(3,2)	(5,3,2,4,1)	(2,3,1)	(2,3,4,5,1)	(3,4,5,2,1)	(3,2,4,5,1)	(4,3,5,2,1)
(5,4,3):	(2,5,1)	(2,4,1)	(3,1)(5,2)	(3,4,2,5,1)	(5,3,4,2,1)	(5,1)(4,2)	(4,5,3,2,1)

	(5,2,1,4,3):	(5,4,1,3,2):	(5,3,2,4,1):	(5,3,1,2,4):	(5,2,4,3,1):
(1,2,3):	(4,1)(5,3)	(5,4,2)	(5,1)(4,2)	(3,2,4,5,1)	(5,3,2,4,1)
(2,3,4):	(2,1)(5,3)	(4,1)(5,2)	(5,4,1)	(3,1)(5,4)	(5,3,1)
(1,3,4):	(3,5,2)	(4,3,2,5,1)	(5,4,3,2,1)	(2,1)(5,4)	(5,2,1)
(1,2,4):	(5,4,3)	(3,4,2,5,1)	(5,3,4,2,1)	(4,5,3,2,1)	(5,4,3,2,1)
(1,2,5):	(4,3,1)	(3,5,4,2,1)	(4,2)(5,3)	(5,3,2,4,1)	(4,3,2)
(1,3,5):	(4,5,2,3,1)	(5,4,3,2,1)	(4,3,2)	(2,4,1)	(4,5,2)
(2,3,5):	(4,5,3,2,1)	(5,4,1)	(2,4,1)	(3,1)(4,2)	(2,4,5,3,1)
(1,4,5):	(5,2,4,3,1)	(3,2,1)	(5,3,2)	(2,5,3,4,1)	(5,2)(4,3)
(2,4,5):	(5,4,3,2,1)	(3,4,1)	(2,5,3,4,1)	(4,2,5,3,1)	(2,5,4,3,1)
(3,4,5):	(5,2,1)	(4,1)(3,2)	(3,2,5,4,1)	(2,5,4,3,1)	(3,1)(5,2)
(3,2,1):	(4,2,3,5,1)	(2,5,4,3,1)	(5,2,4,3,1)	(4,5,2)	(5,1)(4,2)
(4,3,2):	(3,5,4,2,1)	(2,5,3,4,1)	(5,2,3,4,1)	(4,5,2,3,1)	(5,4,2,3,1)
(4,3,1):	(3,5,2,4,1)	(5,3,2)	(5,1)(3,2)	(2,3,4,5,1)	(5,2,3,4,1)
(4,2,1):	(2,4,3,5,1)	(3,1)(5,2)	(5,3,1)	(4,5,3)	(5,1)(4,3)
(5,2,1):	(4,3,2,5,1)	(3,1)(5,4)	(2,4,5,3,1)	(4,2)(5,3)	(2,4,3,5,1)
(5,3,1):	(4,1)(5,2)	(3,2)(5,4)	(3,2,4,5,1)	(2,4,3,5,1)	(3,5,2,4,1)
(5,3,2):	(4,2,1)	(2,3,5,4,1)	(3,5,2,4,1)	(5,2,4,3,1)	(3,1)(4,2)
(5,4,1):	(5,2)(4,3)	(3,2,4,5,1)	(4,5,3,2,1)	(2,1)(5,3)	(4,3,5,2,1)
(5,4,2):	(2,1)(4,3)	(3,5,2,4,1)	(4,1)(5,3)	(5,3,1)	(4,3,1)
(5,4,3):	(3,4,5,2,1)	(5,3,2,4,1)	(4,1)(3,2)	(2,3,1)	(4,5,2,3,1)

GROUP (1,2,3,4,5)
K4*K3

	(1,3,5,2,4):	(1,5,4,2,3):	(1,5,3,4,2):	(1,2,5,4,3):	(1,5,2,3,4):	(1,4,5,3,2):	(5,3,1,4,2):
(1,2,3):	(4,2)(5,3)	(5,4,3,2,1)	(5,1)(4,3)	(3,2,5,4,1)	(5,3,4,2,1)	(4,5,1)	(4,3,2,5,1)
(2,3,4):	(4,1)(5,3)	(5,2,4,3,1)	(5,4,3,2,1)	(3,1)(5,2)	(5,3,2,4,1)	(2,1)(5,4)	(2,5,4,3,1)
(1,3,4):	(4,3,5,2,1)	(5,1)(4,2)	(5,4,2,3,1)	(2,5,1)	(5,2,4,3,1)	(3,2)(5,4)	(5,4,2)
(1,2,4):	(3,5,4,2,1)	(5,1)(3,2)	(5,3,1)	(4,3,2,5,1)	(5,4,2,3,1)	(4,5,3)	(5,3,2)
(1,2,5):	(3,1)(4,2)	(3,2)(5,4)	(4,5,3)	(5,4,3,2,1)	(3,4,2)	(4,1)(5,3)	(4,5,3,2,1)
(1,3,5):	(5,2,4,3,1)	(5,4,2)	(3,4,2)	(2,1)(5,4)	(5,2)(4,3)	(4,1)(3,2)	(4,2,1)
(2,3,5):	(5,3,2,4,1)	(2,5,4,3,1)	(2,1)(4,3)	(3,1)(5,4)	(2,5,3,4,1)	(4,2,1)	(4,3,1)
(1,4,5):	(3,1)(5,2)	(3,4,2)	(4,2)(5,3)	(2,1)(4,3)	(3,5,2)	(5,3,2,4,1)	(5,3,4,2,1)
(2,4,5):	(3,2,5,4,1)	(2,3,1)	(2,1)(5,3)	(4,3,1)	(2,3,5,4,1)	(5,3,4,2,1)	(5,3,1)
(3,4,5):	(4,1)(5,2)	(3,1)(4,2)	(3,5,4,2,1)	(2,3,1)	(3,5,2,4,1)	(5,4,3,2,1)	(5,4,2,3,1)
(3,2,1):	(2,4,3,5,1)	(5,4,1)	(5,2,4,3,1)	(5,4,2)	(5,1)(3,2)	(4,5,2,3,1)	(4,1)(5,2)
(4,3,2):	(2,3,5,4,1)	(5,3,1)	(5,1)(4,2)	(5,4,1)	(5,4,1)	(3,4,5,2,1)	(3,1)(5,2)
(4,3,1):	(3,5,2)	(5,3,4,2,1)	(5,1)(4,2)	(2,5,3,4,1)	(5,2,1)	(3,2,4,5,1)	(3,4,2,5,1)
(4,2,1):	(3,5,1)	(5,2,3,4,1)	(5,3,2,4,1)	(5,2)(4,3)	(5,1)(3,2)	(2,4,5,3,1)	(2,5,3,4,1)
(5,2,1):	(3,2,4,5,1)	(2,3,5,4,1)	(2,5,3,4,1)	(5,4,3)	(2,3,4,5,1)	(4,2,5,3,1)	(4,1)(5,3)
(5,3,1):	(4,5,2)	(3,5,4,2,1)	(3,4,2,5,1)	(2,3,5,4,1)	(3,4,5,2,1)	(5,3,2)	(4,2)(5,3)
(5,3,2):	(2,4,1)	(3,1)(5,4)	(3,4,5,2,1)	(5,2,4,3,1)	(3,4,1)	(4,3,5,2,1)	(4,2,3,5,1)
(5,4,1):	(3,4,5,2,1)	(4,2,3,5,1)	(4,2,5,3,1)	(2,4,3,5,1)	(4,5,2,3,1)	(5,3,2)	(4,2)(5,3)
(5,4,2):	(3,4,1)	(4,5,2,3,1)	(4,5,3,2,1)	(5,2,4,3,1)	(4,1)(3,2)	(2,1)(5,3)	(2,4,5,3,1)
(5,4,3):	(5,2,3,4,1)	(4,2,5,3,1)	(4,2,1)	(2,4,5,3,1)	(4,1)(5,2)	(3,2,1)	(3,1)(4,2)

	(5,4,1,3,2):	(5,1,2,4,3):	(5,2,1,3,4):	(5,1,4,3,2):	(5,4,1,2,3):
(1,2,3):	(5,4,2)	(3,5,2,4,1)	(4,5,3)	(4,1)(5,2)	(3,5,4,2,1)
(2,3,4):	(4,1)(5,2)	(3,5,1)	(4,5,3,2,1)	(2,5,1)	(3,5,2,4,1)
(1,3,4):	(4,3,2,5,1)	(2,1)(5,3)	(4,5,2,3,1)	(5,3,2)	(2,4,3,5,1)
(1,2,4):	(3,4,2,5,1)	(4,3,5,2,1)	(3,1)(5,4)	(5,2)(4,3)	(4,2,3,5,1)
(1,2,5):	(3,5,4,2,1)	(5,2,4,3,1)	(3,4,1)	(4,3,5,2,1)	(5,4,2,3,1)
(1,3,5):	(5,4,3,2,1)	(2,4,5,3,1)	(5,2,3,4,1)	(4,5,3,2,1)	(2,5,4,3,1)
(2,3,5):	(5,4,1)	(3,2,4,5,1)	(5,3,4,2,1)	(4,5,1)	(3,2,5,4,1)
(1,4,5):	(3,2,1)	(4,3,5,2,1)	(5,4,3,2,1)	(5,3,2)	(2,4,3,5,1)
(2,4,5):	(4,3,2,5,1)	(2,3,4,5,1)	(3,1)(5,4)	(2,5,3,4,1)	(5,4,2,3,1)
(3,4,5):	(4,2,1)	(4,3,1)	(2,3,4,5,1)	(3,5,4,2,1)	(4,3,2,5,1)
(3,2,1):	(4,5,3)	(2,4,5,3,1)	(4,5,2,3,1)	(3,4,1)	(4,1)(5,2)
(4,3,2):	(4,5,1)	(2,5,3,4,1)	(4,5,3,2,1)	(4,1)(3,2)	(2,5,3,4,1)
(4,3,1):	(5,3,2)	(2,5,4,3,1)	(4,2,1)	(3,4,2,5,1)	(5,4,3,2,1)
(4,2,1):	(5,3,1)	(2,4,5,3,1)	(4,3,5,2,1)	(3,4,5,2,1)	(5,3,4,2,1)
(5,2,1):	(5,4,1)	(3,4,2,5,1)	(4,3,1)	(3,4,2,5,1)	(5,3,2,4,1)
(5,3,1):	(5,4,2)	(3,5,2,4,1)	(4,5,1)	(3,4,2,5,1)	(5,3,4,2,1)
(5,3,2):	(5,4,1)	(3,5,4,2,1)	(4,5,2)	(3,4,1,5,2)	(5,3,4,2,1)
(5,4,1):	(5,3,2)	(4,2,3,5,1)	(4,5,3,2,1)	(4,1)(3,2)	(5,3,4,2,1)
(5,4,2):	(5,3,1)	(4,2,5,3,1)	(4,2,1)	(2,4,5,3,1)	(5,3,4,2,1)
(5,4,3):	(5,2,1)	(4,2,1)	(4,3,1)	(2,4,5,3,1)	(5,3,4,2,1)

GROUP (1,2,3,4,5)
K4*K5,

	(1,2)(3,4):	(1,3)(2,4):	(1,4)(2,3):	(1,2)(3,5):	(1,3)(2,5):	(1,5)(2,3):	(1,2)(4,5):
(1,2,3):	(3,4,1)	(4,3,2)	(4,2,1)	(3,5,1)	(5,3,2)	(5,2,1)	(3,1)(5,4)
(2,3,4):	(3,2,1)	(4,3,1)	(2,4,1)	(3,5,4,2,1)	(4,2,5,3,1)	(5,1)(4,2)	(3,4,5,2,1)
(1,3,4):	(2,3,1)	(4,2,1)	(4,3,2)	(2,3,5,4,1)	(4,1)(5,2)	(5,3,2,4,1)	(2,3,4,5,1)
(1,2,4):	(4,3,1)	(3,2,1)	(3,4,2)	(4,1)(5,3)	(3,2,5,4,1)	(5,2,3,4,1)	(4,5,1)
(1,2,5):	(5,1)(4,3)	(3,2,4,5,1)	(4,2,3,5,1)	(5,3,1)	(3,2,1)	(3,5,2)	(5,4,1)
(1,3,5):	(2,3,4,5,1)	(5,1)(4,2)	(4,3,2,5,1)	(2,3,1)	(5,2,1)	(5,3,2)	(2,3,5,4,1)
(2,3,5):	(3,4,5,2,1)	(5,2,4,3,1)	(4,1)(5,2)	(3,2,1)	(5,3,1)	(2,5,1)	(3,5,4,2,1)
(1,4,5):	(2,4,3,5,1)	(3,4,2,5,1)	(5,1)(3,2)	(2,4,5,3,1)	(3,4,5,2,1)	(3,2)(5,4)	(2,4,1)
(2,4,5):	(4,3,5,2,1)	(3,1)(5,2)	(5,2,3,4,1)	(4,5,3,2,1)	(3,1)(5,4)	(2,3,4,5,1)	(4,2,1)
(3,4,5):	(2,1)(5,3)	(4,2,5,3,1)	(5,3,2,4,1)	(2,1)(5,4)	(4,5,2,3,1)	(3,2,4,5,1)	(2,1)(4,3)
(3,2,1):	(3,4,2)	(2,4,1)	(4,3,1)	(3,5,2)	(2,5,1)	(5,3,1)	(3,2)(5,4)
(4,3,2):	(4,2,1)	(2,3,1)	(3,4,1)	(4,3,5,2,1)	(2,5,4,3,1)	(5,1)(4,3)	(4,5,3,2,1)
(4,3,1):	(2,4,1)	(3,4,2)	(3,2,1)	(2,4,3,5,1)	(5,2)(4,3)	(5,4,3,2,1)	(2,4,5,3,1)
(4,2,1):	(4,3,2)	(3,4,1)	(2,3,1)	(4,2)(5,3)	(3,4,2,5,1)	(5,4,2,3,1)	(4,5,2)
(5,2,1):	(5,2)(4,3)	(3,5,2,4,1)	(4,5,2,3,1)	(5,3,2)	(3,5,1)	(2,3,1)	(5,4,2)
(5,3,1):	(2,5,3,4,1)	(4,2)(5,3)	(4,5,3,2,1)	(2,5,1)	(3,5,2)	(3,2,1)	(2,5,4,3,1)
(5,3,2):	(5,3,4,2,1)	(2,4,5,3,1)	(4,1)(5,3)	(5,2,1)	(2,3,1)	(3,5,1)	(5,4,3,2,1)
(5,4,1):	(2,5,4,3,1)	(3,5,4,2,1)	(3,2)(5,4)	(2,5,3,4,1)	(3,5,2,4,1)	(4,1)(3,2)	(2,5,1)
(5,4,2):	(5,4,3,2,1)	(3,1)(5,4)	(2,3,5,4,1)	(5,3,4,2,1)	(3,1)(4,2)	(4,2,3,5,1)	(5,2,1)
(5,4,3):	(2,1)(5,4)	(5,4,2,3,1)	(3,2,5,4,1)	(2,1)(4,3)	(5,2,4,3,1)	(4,3,2,5,1)	(2,1)(5,3)

	(1,4)(2,5):	(1,5)(2,4):	(1,3)(4,5):	(1,4)(3,5):	(1,5)(3,4):	(2,3)(4,5):	(2,4)(3,5):
(1,2,3):	(4,2,5,3,1)	(5,2,4,3,1)	(3,2)(5,4)	(4,2,3,5,1)	(5,2,3,4,1)	(2,1)(5,4)	(2,4,3,5,1)
(2,3,4):	(2,5,3,4,1)	(5,1)(4,3)	(4,5,2,3,1)	(2,3,5,4,1)	(5,1)(3,2)	(4,5,2)	(5,4,3)
(1,3,4):	(5,2)(4,3)	(5,3,4,2,1)	(4,5,1)	(5,4,3)	(5,3,1)	(3,2,4,5,1)	(3,5,4,2,1)
(1,2,4):	(5,4,2)	(5,2,1)	(3,2,4,5,1)	(4,2)(5,3)	(5,2,4,3,1)	(2,3,4,5,1)	(2,1)(5,3)
(1,2,5):	(4,2,1)	(4,5,2)	(3,2,5,4,1)	(4,2,5,3,1)	(5,2)(4,3)	(2,3,5,4,1)	(2,4,5,3,1)
(1,3,5):	(4,3,5,2,1)	(4,2)(5,3)	(5,4,1)	(4,3,1)	(4,5,3)	(3,2,5,4,1)	(3,1)(4,2)
(2,3,5):	(4,1)(5,3)	(2,4,3,5,1)	(5,4,2,3,1)	(4,1)(3,2)	(2,3,4,5,1)	(5,4,2)	(4,3,2)
(1,4,5):	(5,2,1)	(5,4,2)	(3,4,1)	(5,3,1)	(5,4,3)	(4,1)(3,2)	(4,2,5,3,1)
(2,4,5):	(5,4,1)	(2,5,1)	(3,1)(4,2)	(5,3,2,4,1)	(2,4,3,5,1)	(3,4,2)	(5,3,2)
(3,4,5):	(5,2,3,4,1)	(3,4,2,5,1)	(4,3,1)	(5,4,1)	(3,5,1)	(4,3,2)	(5,4,2)
(3,2,1):	(4,3,2,5,1)	(5,3,2,4,1)	(2,1)(5,4)	(4,3,5,2,1)	(5,3,4,2,1)	(3,1)(5,4)	(3,5,2,4,1)
(4,3,2):	(3,2,5,4,1)	(5,1)(3,2)	(2,4,5,3,1)	(3,5,2,4,1)	(5,1)(4,2)	(4,5,3)	(3,5,2)
(4,3,1):	(3,1)(5,2)	(5,4,2,3,1)	(4,5,3)	(3,5,1)	(5,4,1)	(4,5,3,2,1)	(4,2,3,5,1)
(4,2,1):	(2,5,1)	(5,4,1)	(3,4,5,2,1)	(2,1)(5,3)	(5,4,3,2,1)	(4,5,2,3,1)	(4,1)(5,3)
(5,2,1):	(4,5,1)	(2,4,1)	(3,5,4,2,1)	(4,5,3,2,1)	(2,1)(4,3)	(5,4,2,3,1)	(5,3,2,4,1)
(5,3,1):	(4,5,2,3,1)	(3,1)(4,2)	(5,4,3)	(4,5,1)	(3,4,1)	(5,4,3,2,1)	(5,1)(4,2)
(5,3,2):	(4,1)(3,2)	(3,2,4,5,1)	(2,5,4,3,1)	(4,1)(5,2)	(3,4,2,5,1)	(5,4,3)	(4,5,2)
(5,4,1):	(4,5,2)	(4,5,1)	(3,5,1)	(4,5,3)	(4,3,1)	(5,1)(3,2)	(5,3,4,2,1)
(5,4,2):	(2,4,1)	(4,2,3,5,1)	(3,1)(5,2)	(2,5,3,4,1)	(4,3,2,5,1)	(3,5,2)	(4,5,3)
(5,4,3):	(3,5,2,4,1)		(5,3,1)	(3,4,1)	(4,5,1)	(5,3,2)	(3,4,2)

	(2,5)(3,4):
(1,2,3):	(2,5,3,4,1)
(2,3,4):	(5,3,2)
(1,3,4):	(3,1)(5,2)
(1,2,4):	(2,5,4,3,1)
(1,2,5):	(2,1)(4,3)
(1,3,5):	(3,4,5,2,1)
(2,3,5):	(4,5,3)
(1,4,5):	(4,3,5,2,1)
(2,4,5):	(5,4,3)
(3,4,5):	(3,5,2)
(3,2,1):	(3,4,2,5,1)
(4,3,2):	(5,4,2)
(4,3,1):	(4,1)(5,2)
(4,2,1):	(4,3,2,5,1)
(5,2,1):	(5,1)(4,3)
(5,3,1):	(5,2,3,4,1)
(5,3,2):	(3,4,2)
(5,4,1):	(5,2,4,3,1)
(5,4,2):	(4,3,2)
(5,4,3):	(4,5,2)

GROUP (1,2,3,4,5)
K5*K2

	(1,2,3,4,5):	(1,2,5,3,4):	(1,4,5,2,3):	(1,4,2,3,5):	(1,3,5,4,2):	(1,3,4,2,5):	(5,4,3,2,1):
(1,2)(3,4):	(4,5,2)	(5,4,2)	(3,2,4,5,1)	(3,5,2,4,1)	(4,1)(5,3)	(4,1)(5,2)	(5,3,1)
(1,3)(2,4):	(4,5,3,2,1)	(4,3,2,5,1)	(2,1)(5,4)	(2,1)(5,3)	(3,5,2)	(5,3,2)	(5,2,3,4,1)
(1,4)(2,3):	(3,1)(5,4)	(3,1)(5,2)	(4,5,3)	(5,4,3)	(2,4,3,5,1)	(2,5,4,3,1)	(5,1)(4,2)
(1,2)(3,5):	(5,2)(4,3)	(3,4,2)	(4,3,2,5,1)	(4,1)(5,2)	(5,4,1)	(5,2,3,4,1)	(3,1)(5,4)
(1,3)(2,5):	(5,3,4,2,1)	(5,1)(4,3)	(4,2,1)	(4,5,3,2,1)	(3,2)(5,4)	(4,5,3)	(2,3,5,4,1)
(1,5)(2,3):	(3,4,1)	(3,4,5,2,1)	(4,1)(5,3)	(4,3,1)	(2,5,4,3,1)	(2,1)(4,3)	(5,4,2)
(1,2)(4,5):	(3,5,2)	(4,2)(5,3)	(5,1)(3,2)	(5,2,3,4,1)	(3,4,1)	(3,5,2,4,1)	(4,3,1)
(1,4)(2,5):	(5,4,2,3,1)	(5,3,1)	(3,4,2)	(3,2)(5,4)	(3,2,4,5,1)	(3,1)(5,4)	(2,4,3,5,1)
(1,5)(2,4):	(4,1)(3,2)	(4,5,3,2,1)	(2,3,5,4,1)	(2,3,1)	(3,1)(5,2)	(3,2,1)	(5,2)(4,3)
(1,3)(4,5):	(2,1)(5,3)	(2,4,3,5,1)	(5,2,1)	(5,3,4,2,1)	(3,4,2)	(4,2)(5,3)	(4,1)(3,2)
(1,4)(3,5):	(2,5,4,3,1)	(2,3,1)	(5,2)(4,3)	(5,4,2)	(5,1)(4,2)	(5,4,2,3,1)	(3,2,4,5,1)
(1,5)(3,4):	(2,4,1)	(2,1)(5,4)	(3,5,2,4,1)	(3,1)(4,2)	(4,2,5,3,1)	(4,2,1)	(5,3,2)
(2,3)(4,5):	(3,5,1)	(3,5,2,4,1)	(5,3,1)	(5,1)(4,3)	(2,1)(4,3)	(2,4,3,5,1)	(4,2,1)
(2,4)(3,5):	(4,3,2,5,1)	(4,1)(3,2)	(2,5,4,3,1)	(2,5,1)	(5,2,1)	(5,1)(3,2)	(3,4,5,2,1)
(2,5)(3,4):	(5,1)(4,2)	(5,4,1)	(3,1)(4,2)	(3,2,4,5,1)	(4,5,3,2,1)	(4,5,1)	(2,1)(5,3)

	(5,2,1,4,3):	(5,4,1,3,2):	(5,3,2,4,1):	(5,3,1,2,4):	(5,2,4,3,1):
(1,2)(3,4):	(3,5,1)	(4,2,5,3,1)	(5,4,2,3,1)	(3,2)(5,4)	(5,1)(3,2)
(1,3)(2,4):	(2,3,5,4,1)	(5,2)(4,3)	(5,1)(4,3)	(4,5,1)	(5,4,1)
(1,4)(2,3):	(4,2)(5,3)	(2,5,1)	(5,2,1)	(3,4,5,2,1)	(5,3,4,2,1)
(1,2)(3,5):	(4,5,1)	(5,4,2,3,1)	(3,1)(4,2)	(4,3,2)	(3,2,4,5,1)
(1,3)(2,5):	(4,1)(3,2)	(5,4,3)	(2,4,3,5,1)	(5,1)(4,2)	(2,4,1)
(1,5)(2,3):	(4,2,5,3,1)	(2,1)(5,4)	(4,5,2)	(3,5,2,4,1)	(4,2)(5,3)
(1,2)(4,5):	(5,1)(4,3)	(3,1)(4,2)	(4,2,5,3,1)	(5,3,2)	(4,3,2,5,1)
(1,4)(2,5):	(4,3,2)	(3,5,1)	(2,1)(5,3)	(5,3,4,2,1)	(2,1)(4,3)
(1,5)(2,4):	(2,5,4,3,1)	(3,4,5,2,1)	(4,5,3)	(4,1)(5,3)	(5,4,3)
(1,3)(4,5):	(5,2,3,4,1)	(4,3,2)	(4,3,2,5,1)	(2,5,1)	(4,1)(5,2)
(1,4)(3,5):	(4,5,2)	(5,1)(3,2)	(3,2,1)	(2,1)(4,3)	(3,4,5,2,1)
(1,5)(3,4):	(3,1)(5,2)	(4,5,3,2,1)	(3,2)(5,4)	(2,3,5,4,1)	(3,5,2)
(2,3)(4,5):	(5,3,4,2,1)	(2,4,1)	(4,1)(5,2)	(3,1)(5,2)	(4,2,5,3,1)
(2,4)(3,5):	(2,1)(5,4)	(5,2,3,4,1)	(3,4,1)	(4,3,1)	(3,1)(5,4)
(2,5)(3,4):	(3,2,1)	(4,1)(5,3)	(2,3,5,4,1)	(5,4,2,3,1)	(2,3,1)

GROUP (1,2,3,4,5)
K5*K3

	(1,3,5,2,4):	(1,5,4,2,3):	(1,5,3,4,2):	(1,2,5,4,3):	(1,5,2,3,4):	(1,4,5,3,2):	(5,3,1,4,2):
(1,2)(3,4):	(4,2,3,5,1)	(5,3,2,4,1)	(5,4,1)	(5,3,2)	(5,1)(4,2)	(3,1)(5,4)	(3,2,5,4,1)
(1,3)(2,4):	(5,4,3)	(5,2,1)	(5,1)(3,2)	(4,1)(5,2)	(5,4,3,2,1)	(2,3,4,5,1)	(2,5,1)
(1,4)(2,3):	(2,1)(5,3)	(5,1)(4,3)	(5,2,4,3,1)	(3,4,2,5,1)	(5,3,1)	(4,5,2)	(5,2)(4,3)
(1,2)(3,5):	(5,1)(4,2)	(3,2,5,4,1)	(3,4,1)	(3,2)(5,4)	(3,4,2,5,1)	(4,3,1)	(4,1)(3,2)
(1,3)(2,5):	(4,3,2)	(2,1)(5,4)	(2,3,4,5,1)	(5,4,1)	(2,1)(4,3)	(4,2,3,5,1)	(4,5,1)
(1,5)(2,3):	(2,4,5,3,1)	(5,4,3)	(5,2)(4,3)	(3,5,4,2,1)	(4,5,3)	(4,1)(5,2)	(4,3,5,2,1)
(1,2)(4,5):	(3,4,2,5,1)	(4,1)(3,2)	(4,1)(5,3)	(4,3,2)	(4,2,3,5,1)	(5,3,1)	(5,3,2,4,1)
(1,4)(2,5):	(3,2,1)	(2,3,4,5,1)	(2,4,5,3,1)	(5,1)(4,3)	(2,3,1)	(4,2)(5,3)	(4,5,3)
(1,5)(2,4):	(3,1)(5,4)	(3,5,2)	(5,3,2)	(4,3,5,2,1)	(3,2)(5,4)	(2,5,3,4,1)	(2,1)(5,3)
(1,3)(4,5):	(5,2)(4,3)	(4,2,1)	(4,2,3,5,1)	(2,4,1)	(4,3,5,2,1)	(5,1)(3,2)	(5,1)(4,2)
(1,4)(3,5):	(5,2,1)	(3,4,2,5,1)	(3,1)(4,2)	(2,3,4,5,1)	(3,1)(5,2)	(4,3,2)	(3,4,2)
(1,5)(3,4):	(4,5,2,3,1)	(4,2)(5,3)	(5,4,2)	(2,1)(5,3)	(4,5,2)	(3,2,5,4,1)	(3,5,4,2,1)
(2,3)(4,5):	(2,5,3,4,1)	(4,3,1)	(4,3,5,2,1)	(3,1)(4,2)	(4,1)(5,3)	(5,2,1)	(5,2,4,3,1)
(2,4)(3,5):	(5,4,1)	(3,1)(5,2)	(3,2,1)	(4,5,2,3,1)	(3,2,5,4,1)	(2,1)(4,3)	(2,3,1)
(2,5)(3,4):	(4,1)(3,2)	(2,4,5,3,1)	(2,1)(5,4)	(5,3,1)	(2,4,1)	(3,5,4,2,1)	(3,1)(5,4)

	(5,4,1,3,2):	(5,1,2,4,3):	(5,2,1,3,4):	(5,1,4,3,2):	(5,4,1,2,3):
(1,2)(3,4):	(4,2,5,3,1)	(3,5,2)	(4,5,1)	(3,1)(5,2)	(4,2)(5,3)
(1,3)(2,4):	(5,2)(4,3)	(4,1)(5,3)	(3,2)(5,4)	(2,5,3,4,1)	(4,3,5,2,1)
(1,4)(2,3):	(2,5,1)	(3,5,4,2,1)	(2,4,5,3,1)	(5,4,2)	(3,5,1)
(1,2)(3,5):	(5,4,2,3,1)	(4,5,2)	(5,1)(4,3)	(4,5,2,3,1)	(5,4,2)
(1,3)(2,5):	(5,4,3)	(5,3,2,4,1)	(3,4,2)	(4,1)(5,3)	(5,4,3,2,1)
(1,5)(2,3):	(2,1)(5,4)	(3,1)(4,2)	(2,5,3,4,1)	(4,2,1)	(3,1)(5,4)
(1,2)(4,5):	(3,1)(4,2)	(5,2)(4,3)	(3,5,1)	(5,2,4,3,1)	(3,4,2)
(1,4)(2,5):	(3,5,1)	(5,4,3,2,1)	(3,1)(4,2)	(5,4,3)	(5,1)(3,2)
(1,5)(2,4):	(3,4,5,2,1)	(4,3,1)	(3,2,5,4,1)	(2,1)(4,3)	(4,5,2,3,1)
(1,3)(4,5):	(4,3,2)	(2,5,3,4,1)	(3,5,2)	(5,3,2,4,1)	(2,1)(4,3)
(1,4)(3,5):	(5,1)(3,2)	(2,1)(5,4)	(5,2,4,3,1)	(3,2)(5,4)	(2,5,1)
(1,5)(3,4):	(4,5,3,2,1)	(2,3,1)	(4,1)(5,2)	(3,2,1)	(2,4,5,3,1)
(2,3)(4,5):	(2,4,1)	(3,4,2,5,1)	(2,1)(5,3)	(5,1)(4,2)	(3,4,1)
(2,4)(3,5):	(5,2,3,4,1)	(4,5,1)	(5,4,3,2,1)	(5,4,3,2,1)	(4,1)(5,2)
(2,5)(3,4):	(4,1)(5,3)	(5,1)(3,2)	(4,2,1)	(3,5,1)	(5,3,2,4,1)

GROUP (1,2,3,4,5)
K5*K4

	(1,2,3):	(2,3,4):	(1,3,4):	(1,2,4):	(1,2,5):	(1,3,5):	(2,3,5):
(1,2)(3,4):	(4,3,2)	(2,4,1)	(4,2,1)	(3,4,2)	(5,2)(4,3)	(4,3,5,2,1)	(2,4,3,5,1)
(1,3)(2,4):	(4,2,1)	(3,2,1)	(4,3,2)	(4,3,1)	(4,2,5,3,1)	(4,2)(5,3)	(3,5,4,2,1)
(1,4)(2,3):	(3,4,1)	(4,3,1)	(2,3,1)	(3,2,1)	(3,2,5,4,1)	(2,3,5,4,1)	(4,1)(5,3)
(1,2)(3,5):	(5,3,2)	(2,5,3,4,1)	(5,3,4,2,1)	(4,2)(5,3)	(3,5,2)	(5,2,1)	(2,5,1)
(1,3)(2,5):	(5,2,1)	(3,4,5,2,1)	(5,2)(4,3)	(5,2,4,3,1)	(5,3,1)	(5,3,2)	(3,2,1)
(1,5)(2,3):	(3,5,1)	(5,1)(4,3)	(2,3,4,5,1)	(3,2,4,5,1)	(3,2,1)	(2,3,1)	(5,3,1)
(1,2)(4,5):	(3,2)(5,4)	(2,3,5,4,1)	(3,5,4,2,1)	(5,4,2)	(4,5,2)	(3,4,5,2,1)	(2,3,4,5,1)
(1,4)(2,5):	(5,2,3,4,1)	(4,5,2,3,1)	(3,1)(5,2)	(5,2,1)	(5,4,1)	(3,2,5,4,1)	(4,1)(3,2)
(1,5)(2,4):	(4,2,3,5,1)	(5,1)(3,2)	(3,2,4,5,1)	(4,5,1)	(4,2,1)	(3,1)(4,2)	(5,4,2,3,1)
(1,3)(4,5):	(2,1)(5,4)	(3,5,4,2,1)	(5,4,3)	(2,5,4,3,1)	(2,4,5,3,1)	(4,5,3)	(3,4,5,2,1)
(1,4)(3,5):	(2,5,3,4,1)	(4,2,5,3,1)	(5,3,1)	(2,1)(5,3)	(2,3,5,4,1)	(5,4,1)	(4,1)(5,2)
(1,5)(3,4):	(2,4,3,5,1)	(5,1)(4,2)	(4,5,1)	(2,3,4,5,1)	(2,1)(4,3)	(4,3,1)	(5,2,4,3,1)
(2,3)(4,5):	(3,1)(5,4)	(5,4,3)	(2,3,5,4,1)	(3,2,5,4,1)	(3,2,4,5,1)	(2,3,4,5,1)	(4,5,3)
(2,4)(3,5):	(4,2,5,3,1)	(5,3,2)	(5,3,2,4,1)	(4,1)(5,3)	(4,2,3,5,1)	(5,1)(4,2)	(5,4,2)
(2,5)(3,4):	(5,2,4,3,1)	(4,5,2)	(4,1)(5,2)	(5,2,3,4,1)	(5,1)(4,3)	(4,3,2,5,1)	(4,3,2)

	(1,4,5):	(2,4,5):	(3,4,5):	(3,2,1):	(4,3,2):	(4,3,1):	(4,2,1):
(1,2)(3,4):	(3,4,5,2,1)	(2,3,4,5,1)	(2,1)(5,4)	(4,3,1)	(2,3,1)	(3,2,1)	(3,4,1)
(1,3)(2,4):	(2,4,5,3,1)	(3,1)(5,4)	(3,2,4,5,1)	(3,4,2)	(3,4,1)	(2,4,1)	(2,3,1)
(1,4)(2,3):	(3,2)(5,4)	(4,5,3,2,1)	(4,5,2,3,1)	(2,4,1)	(4,2,1)	(3,4,2)	(4,3,2)
(1,2)(3,5):	(4,3,5,2,1)	(2,4,3,5,1)	(2,1)(4,3)	(5,3,1)	(2,4,5,3,1)	(4,5,3,2,1)	(4,1)(5,3)
(1,3)(2,5):	(4,2,5,3,1)	(3,1)(4,2)	(3,4,2,5,1)	(3,5,2)	(3,5,2,4,1)	(4,1)(5,2)	(4,5,2,3,1)
(1,5)(2,3):	(4,1)(3,2)	(5,3,2,4,1)	(5,2,3,4,1)	(2,5,1)	(5,1)(4,2)	(4,2,3,5,1)	(4,3,2,5,1)
(1,2)(4,5):	(5,2,1)	(2,5,1)	(2,1)(5,3)	(3,1)(5,4)	(2,5,4,3,1)	(5,4,3,2,1)	(5,4,1)
(1,4)(2,5):	(5,4,2)	(4,2,1)	(4,2,5,3,1)	(3,5,2,4,1)	(4,3,5,2,1)	(5,2)(4,3)	(4,5,2)
(1,5)(2,4):	(2,4,1)	(5,4,1)	(5,3,2,4,1)	(3,4,2,5,1)	(5,1)(4,3)	(2,4,3,5,1)	(2,5,1)
(1,3)(4,5):	(5,3,1)	(3,1)(5,2)	(3,5,1)	(3,2)(5,4)	(3,2,5,4,1)	(5,4,1)	(5,4,2,3,1)
(1,4)(3,5):	(5,4,3)	(4,3,5,2,1)	(4,3,1)	(5,3,2,4,1)	(4,5,3,2,1)	(4,5,3)	(4,2)(5,3)
(1,5)(3,4):	(3,4,1)	(5,2,3,4,1)	(5,4,1)	(4,3,2,5,1)	(5,1)(3,2)	(3,5,1)	(3,4,2,5,1)
(2,3)(4,5):	(5,1)(3,2)	(5,3,2)	(3,5,2)	(2,1)(5,4)	(5,4,2)	(5,4,2,3,1)	(5,4,3,2,1)
(2,4)(3,5):	(2,4,3,5,1)	(5,4,3)	(4,3,2)	(5,3,4,2,1)	(4,5,3)	(2,4,5,3,1)	(2,1)(5,3)
(2,5)(3,4):	(3,4,2,5,1)	(3,4,2)	(5,4,2)	(4,3,5,2,1)	(3,5,2)	(3,1)(5,2)	(3,4,5,2,1)

	(5,2,1):	(5,3,1):	(5,3,2):	(5,4,1):	(5,4,2):	(5,4,3):
(1,2)(3,4):	(5,1)(4,3)	(5,4,3,2,1)	(2,5,4,3,1)	(5,3,4,2,1)	(2,5,3,4,1)	(2,1)(5,3)
(1,3)(2,4):	(5,4,2,3,1)	(5,1)(4,2)	(3,4,2,5,1)	(5,2,4,3,1)	(3,1)(5,2)	(3,5,2,4,1)
(1,4)(2,3):	(5,3,2,4,1)	(5,2,3,4,1)	(4,1)(5,2)	(5,1)(3,2)	(4,3,2,5,1)	(4,2,3,5,1)
(1,2)(3,5):	(3,5,1)	(3,2,1)	(2,3,1)	(3,5,4,2,1)	(2,3,5,4,1)	(2,1)(5,4)
(1,3)(2,5):	(2,3,1)	(2,5,1)	(3,5,1)	(2,5,4,3,1)	(3,1)(5,4)	(3,2,5,4,1)
(1,5)(2,3):	(5,3,2)	(3,5,2)	(5,2,1)	(3,2)(5,4)	(5,4,3,2,1)	(5,4,2,3,1)
(1,2)(4,5):	(4,5,1)	(4,5,3,2,1)	(2,4,5,3,1)	(4,2,1)	(2,4,1)	(2,1)(4,3)
(1,4)(2,5):	(2,4,1)	(2,5,3,4,1)	(4,1)(5,3)	(2,5,1)	(4,5,1)	(4,3,2,5,1)
(1,5)(2,4):	(5,4,2)	(4,2)(5,3)	(5,3,4,2,1)	(4,5,2)	(5,2,1)	(5,2,4,3,1)
(1,3)(4,5):	(4,5,2,3,1)	(4,5,1)	(3,2,4,5,1)	(4,3,1)	(3,1)(4,2)	(3,4,1)
(1,4)(3,5):	(3,5,2,4,1)	(3,4,1)	(4,1)(3,2)	(3,5,1)	(4,2,3,5,1)	(4,5,1)
(1,5)(3,4):	(5,2)(4,3)	(5,4,3)	(5,4,3,2,1)	(4,5,3)	(5,3,4,2,1)	(5,3,1)
(2,3)(4,5):	(4,5,3,2,1)	(4,5,2,3,1)	(4,5,2)	(4,1)(3,2)	(3,5,2,4,1)	(4,5,3)
(2,4)(3,5):	(3,5,4,2,1)	(3,1)(4,2)	(3,4,2)	(3,5,2,4,1)	(3,5,2)	(4,5,2)
(2,5)(3,4):	(2,1)(4,3)	(2,5,4,3,1)	(5,4,3)	(2,5,3,4,1)	(4,5,3)	(5,3,2)

PROGRAM DESCRIPTION

General Description

The program computes a group table for elements of S_n which are read in as a series of column and a series of row operations in cycle notation. The general procedure consists of the following steps:

(1) A set of K column operations and L row operations is read into the program.

(2) Each of these operations is broken down into its equivalent sequence of transpositions. Let the group operation P_{ij} represent the (group) product of the ith row operation R_i by the jth column operation C_j. Then P_{ij} will be stored as a long sequence of transpositions.

(3) The SC is rearranged in accordance with the sequence of transpositions which represent P_{ij}.

(4) The rearrangement is examined and a single group operation which effects the same rearrangement of SC is identified as the product $R_i * C_j$.

(5) This single group operation is entered in the ith row and j^{th} column of the group table array.

The manner in which the machine performs steps (2) to (5) is far from standard. For this reason, an illustrative example is carried through in some detail. The example chosen is the entry in A_5 corresponding to the group multiplication (14235) (521).

For future reference, the notation {SYMBOL(-, N, M)} is used to denote a complete set of entries for the triply subscripted quantity SYMBOL(L, N, M) over the full range of the first subscript L for fixed N and M. Thus, {ANS(-, I1, I2)} is the collection of ANS(I, I1, I2) values with $I1^{th}$ column and $I2^{th}$ row fixed and with I allowed to range from its minimum to its maximum value.

The notation $[A_1, A_2, \ldots, A_n]$ is used to display the actual contents of an array. This notation signifies that the content of the first storage location of the array is A_1, that of the second storage location is A_2, etc.

An outline of the program, broken down into sections which perform recognizable functions, follows. (The flow charts and listings in appendixes A and B contain more detailed descriptions.)

Block 1 - set up constants. - The constants used in the program are given literal names and are declared either INTEGER or LOGICAL in TYPE statements. All the subscripted variables are dimensioned and allocated storage locations by the use of a DIMENSION statement. A DATA statement is used to give literal names to the following variables:

Variable	Literal name	Identification
(	LP	Left parenthesis
)	RP	Right parenthesis
,	CM	Comma
	BLK	Blank
E	IDNT	Identity element of group

Block 2 - read in DATA. - The data describing the group and the group operations are read into the program. The labels to be assigned to the objects and to the locations are read from the list for IDENT which is limited to one card. It is read in with FORMAT(80A1) and stored in INITAL(N) in the form bbbX, where b represents an 8-bit blank and X represents the 8-bit location in the 32-bit storage register in which the integer from IDENT is stored. This storing arrangement is accomplished by subroutine SQUEZ, which is described later.

Description of Input

The input cards are read in the following order:

First card	TITLE	one card with FORMAT(80A1)
Second card	IDENT	one card with FORMAT(80A1)

Note that the input for IDENT must be long enough to include the maximum number of objects to be moved in any group operation. Thus, the list for IDENT must be of the form $(A_1, A_2, \ldots A_n)$, where n is the subscript in S_n and A_i is any alphanumeric symbol.

The next set of cards contains the ACROSS (column) operations. There can be as many as four cards with FORMAT(80A1), and each operation is followed by a colon.

A blank card follows the preceding set. It is needed to signal the end of the ACROSS input.

The next set contains DOWN (row) operations. There can be as many as four cards with FORMAT(80A1), and again each operation is followed by a colon.

A blank card follows the preceding set. It is needed to signal the end of the DOWN input.

The input for both ACROSS and DOWN operations is in cycle notation.

Several error checks are run on IDENT. These include checking the maximum number of nonblank units (which is six in this version of the program), the presence of

blank spaces on the card, and the illegal use of parentheses.

The group elements themselves are read from the list for INPUT(J, K), which is also read in with FORMAT(80A1). The complete set of column operations is read first. The upper limit of this input is four cards followed by a blank card. Then the complete set of row operations is read in. This set is also limited to four cards followed by a blank card.

Example of Input - Class Algebra Block $K_2 * K_4$ of A_5

```
     K2*K4
 (1,2,3,4,5)
(1,2,3):(2,3,4):(1,3,4):(1,2,4):(1,2,5):(1,3,5):(2,3,5):(1,4,5):(2,4,5):(3,4,5):
(3,2,1):(4,3,2):(4,3,1):(4,2,1):(5,2,1):(5,3,1):(5,3,2):(5,4,1):(5,4,2):(5,4,3):
Card 5 is blank.
 (1,2,3,4,5):(1,2,5,3,4):(1,4,5,2,3):(1,4,2,3,5):(1,3,5,4,2):(1,3,4,2,5):
 (5,4,3,2,1):(5,2,1,4,3):(5,4,1,3,2):(5,3,2,4,1):(5,3,1,2,4):(5,2,4,3,1):
Card 8 is blank.
```

Cards 3 and 4 are ACROSS (column) operations, and cards 6 and 7 are DOWN (row) operations.

Block 3 - store each group operation as product of transpositions. - In this block, each group operation is decomposed into its equivalent product of transpositions and then stored in PAIRS(I, NOP, K) (see appendix B). The 3rd subscript K is 1 or 2 according to whether the operation is a column or row element, respectively. Thus, an entry in PAIRS(I, NOP, K) is the Ith unit of the NOP^{th} column (K = 1) or row (K = 2) group element.

The example used herein is an entry in the $K_2 * K_4$ class product block of A_5. The input for this block, which was given in the preceding section, shows that the element (521) was the 15th ACROSS (column) operation and therefore corresponds to K = 1, NOP = 15. Before being stored in a PAIRS array, the element is considered to be of the form (52)(21). The contents of {PAIRS(-, 15, 1)} is [5 2 2 1] or

PAIRS(1, 15, 1)	PAIRS(2, 15, 1)	PAIRS(3, 15, 1)	PAIRS(4, 15, 1)
5	2	2	1

The element (14235) is the 4th DOWN (row) element, and it therefore corresponds to K = 2, NOP = 4. The contents of the eight registers which constitute the complete {PAIRS(-, 4, 2)} array are [1 4 4 2 2 3 3 5].

Block 4 - set up arrays in standard configuration. - In this block, a number of standard configurations are set up. If N_C and N_R are the total number of column and row operations, respectively, then $N_C \times N_R$ SC's are set up. A given SC is set up by storing the quantity bbbI in location ANS(I, II, I2) (see appendix B). For a fixed I1

and I2, and SC is the complete set of ANS(I, I1, I2) over the full range of I (see example in block 5).

It is very important to note the differences in the roles of the second and third subscripts here and in PAIRS. In ANS, the second subscript always denotes the $I1^{th}$ colum entry and the third subscript always denotes the $I2^{th}$ row entry. In PAIRS, on the other hand, the third subscript is used to denote whether a row (K = 2) or a column (K = 1) operation is involved. The second subscript labels which row or column. However, an entry in PAIRS never mixes row and column operations: it refers either to one or to the other, depending on whether the third subscript is 1 or 2.

Block 5 - perform group multiplication. - In this block, the indicated operations for all the group products operation I2 * operation I1 (= $P_{I2, I1}$) are actually performed. The method used is to transpose successively units from the SC of {ANS(-, I1, I2)} in accordance with the indicated operation of {PAIRS(-, I1, 1)} followed by the operation of {PAIRS(-, I2, 2)}. The way in which this is accomplished may be understood by following the procedure on a single, complete {ANS(-, I1, I2)} array.

The storage location ascribed to ANS(J, I1, I2) is considered to be the Jth location of the SC. The quantity stored in ANS(J, I1, I2) is considered to be the "thing" which moved to the Jth location as a result of the operation (operation I2 * operation I1) on the SC. Schematically, upon entering block 5, the {ANS(-, I1, I2)} array is in the SC. Thus, for A_5, {ANS(-, I1, I2)} = [1 2 3 4 5].

Continuing with the example of (14235)(521), when K = 1, I1 = 15, and when K = 2, I2 = 4. Therefore, {ANS(-, I1, I2)} = {ANS(-, 15, 4)}. The first rearrangement of the SC of this ANS array is the one effected by {PAIRS(-, 15, 1)}. After being operated on by PAIRS(-, 15, 1), the {ANS(-, 15, 4)} array is [5 1 3 4 2]. After this rearrangement by {PAIRS(-, 15, 1)}, the $I2^{th}$ row operation {PAIRS(-, 4, 2)} is brought into play. After the two successive rearrangements effected by the two PAIRS arrays, the {ANS(-, 15, 4)} array is [1 4 5 2 3]. This is the content of {ANS(-, 15, 4)} upon leaving block 5 and entering block 6.

Block 6 - identification of single group product element. - In effect, the permuted stored values in each ANS array leaving block 5 are examined and a determination is made of the single group operation which would have permuted the SC to this final ANS array in one step. It should be recalled that the permuted ANS array which enters block 6 is the result of the successive operations - operation I1 acting on the SC, followed by operation I2 acting on this result.

The same final ANS array can always be obtained by the action of a single group operation on the SC. In block 6, this single operation is determined and stored in {OUT(-, I1, I2)}. Thus, an array {OUT(-, I1, I2)} represents the single group operation which has the same effect on the standard ANS array (SC) as does the product of the two operations (operation I2 * operation I1).

In order to understand how the program makes the identification, reference may be made to the contents of the {ANS(-, 15, 4)} array upon entering block 6. The content of a given register of ANS is the same as the label of the location in the SC. Since, in the SC, the location and the object had the same label, an examination of the content of a particular ANS register provides partial information about the rearrangement of two objects. Referring to the {ANS(-, 15, 4)} array shows the content of register (2, 15, 4) to be the integer 4. This means that object 4 now occupies the position originally held by object 2. In cycle notation, this state is indicated by placing a 2 to the left of 4 - thus, 2, 4.

The next step is to examine the contents of register 4. Since a 2 is found there, it is clear that a simple transposition of objects 2 and 4 has occurred, so that one part of the group element is the two unit cycle (24).

An examination of the content of register 3 will show a 5. This then leads to an examination of register 5. The content of register 5 is 3, so that another complete cycle has been found. An examination of register 1 (which is actually made first) shows a 1 to be there. This fact is correctly interpreted as evidence that object 1 has not moved, and no 1 will appear in the group element. The examination of all the registers of {ANS(-, 15, 4)} is now complete and the conclusion is reached that the two operation (521) followed by (14235) is equivalent to the single operation (42)(53). Therefore, OUT(-, 15, 4) = (4, 2)(5, 3). This agrees with the entry in the table at the intersection of the column headed by (521) and the row headed by (14235).

It would be confusing to examine group table which contained cycles representing the same group operation but written in a different way. Such cycles are called equivalent cycles. A trivial example would be a transposition. Thus, (24) and (42) certainly represent the same operation. Less trivial examples are (123) and its cyclic permatation (231) as well as (312). It should be noted that equivalent cycles cannot be constructed in this program. To understand this, it is sufficient to note two facts. First, the configuration resulting from the operation of a given element of S_n on the SC is unique. Since equivalent cycles represent the same group element, they all result in the same unique final array. Therefore, two final ANS arrays leaving block 5 can only differ from one another if they are associated with two distinct group elements. Secondly, block 6 follows the same procedure in examining each ANS array and in storing the findings in the corresponding OUT array. Therefore, if {ANS(-, I1, I2)} has the same content as {ANS(-, I3, I4)}, then {OUT(-, I1, I2)} is identical to {OUT(-, I3, I4)}.

Block 7 - output and error messages. - In this block, the group table is listed. The error messages are also contained here.

Subroutine SQUEZ. - The usual manner of storing data on the IBM 360 series is in a word of 32 bits. SQUEZ packs four pieces of data in one storage word. It also elimi-

nates extraneous blanks. Before calling SQUEZ an entry in the group multiplication table would look like the following:

bbb(bbb1bbb, bbb2bbb, bbb3bbb)

After calling SQUEZ we have the following:

(1, 2, 3)

SUMMARY OF PROCEDURE

A FORTRAN program for use on the IBM 360-67 has been written for the computation of group tables for finite groups. Use is made of the existence of an ismorphism between any finite group of order n and some subgroup of the symmetric group S_n. Those elements of S_n which are isomorphic to the group of interest are fed into the program as input data. The program then performs the group operations on these elements of S_n and identifies the products of these operations.

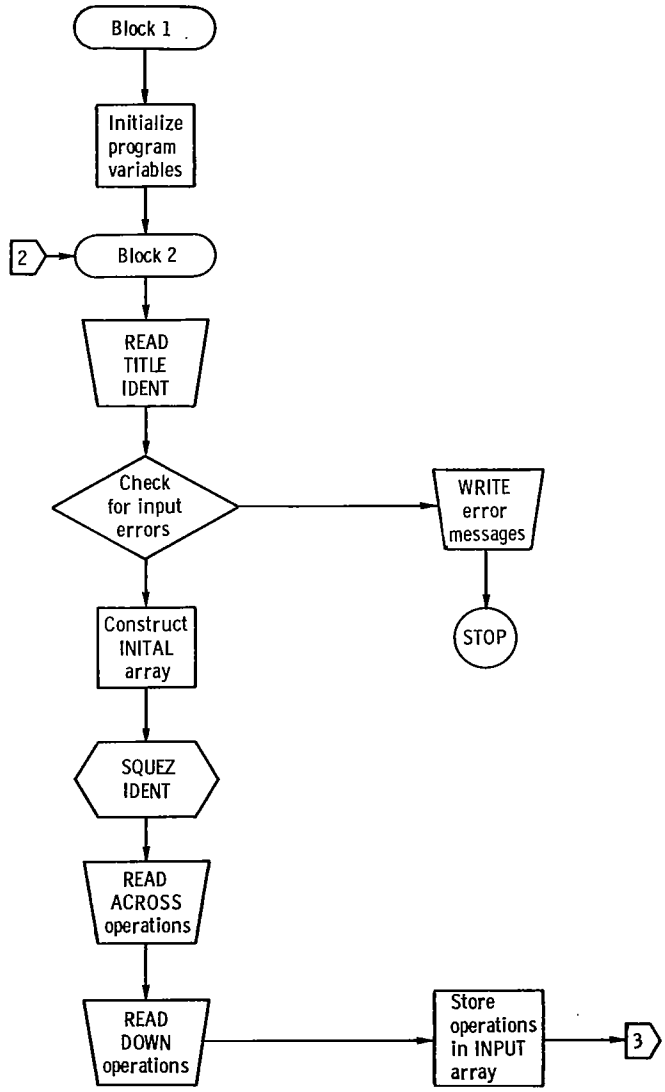
The procedure is based on the fact that every element of S_n can be expressed as a product of transpositions or simple interchanges of two objects. The program expresses each element given as input data in this form and performs the indicated interchanges for a group product of two such elements on a standrad configuration. The resulting configuration is then identified with that configuration which a single group element would cause if applied to the standard configuration. This group element is called the product of the other two and entered in the row and column headed by the original two elements of S_n.

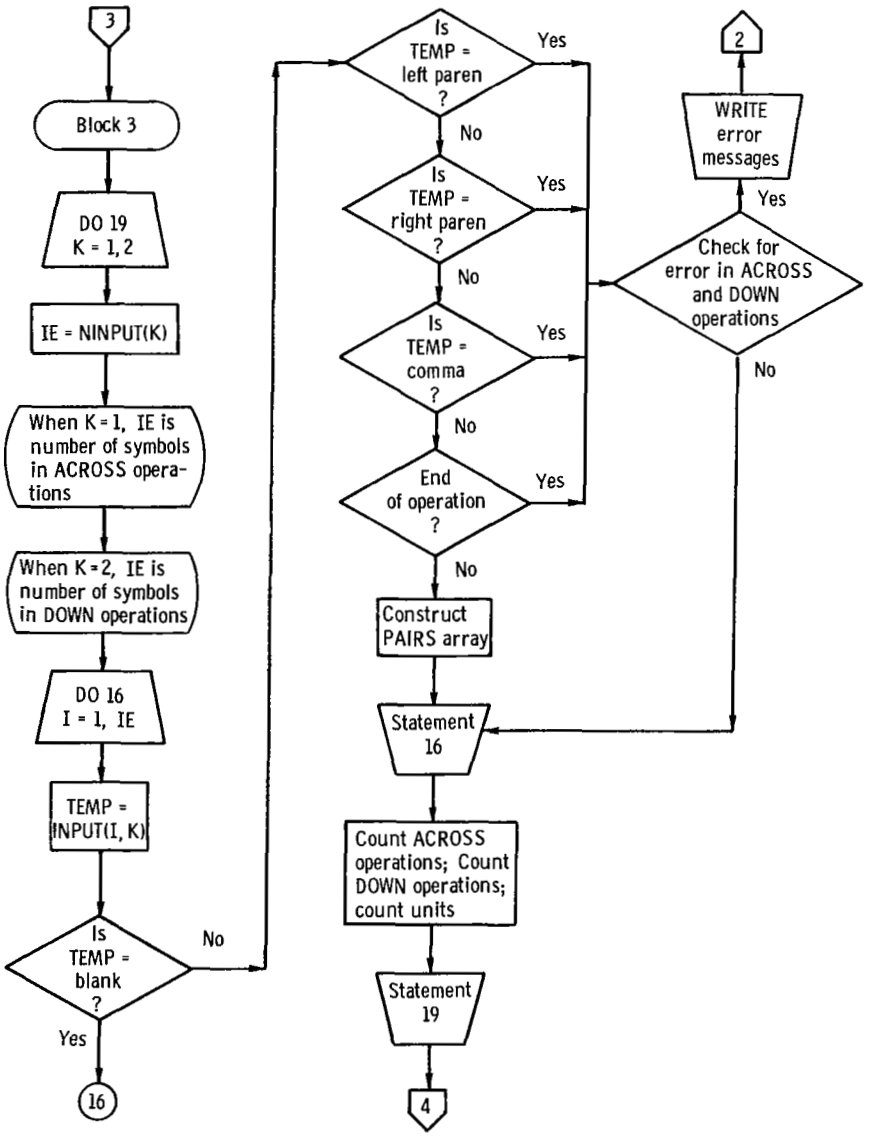
Two examples are shown worked out in the report. One of these is the complete table of S_4, the other is A_5. For the latter, the class algebra table is presented. This table, which is of interest in itself, does not appear to have been previously published.

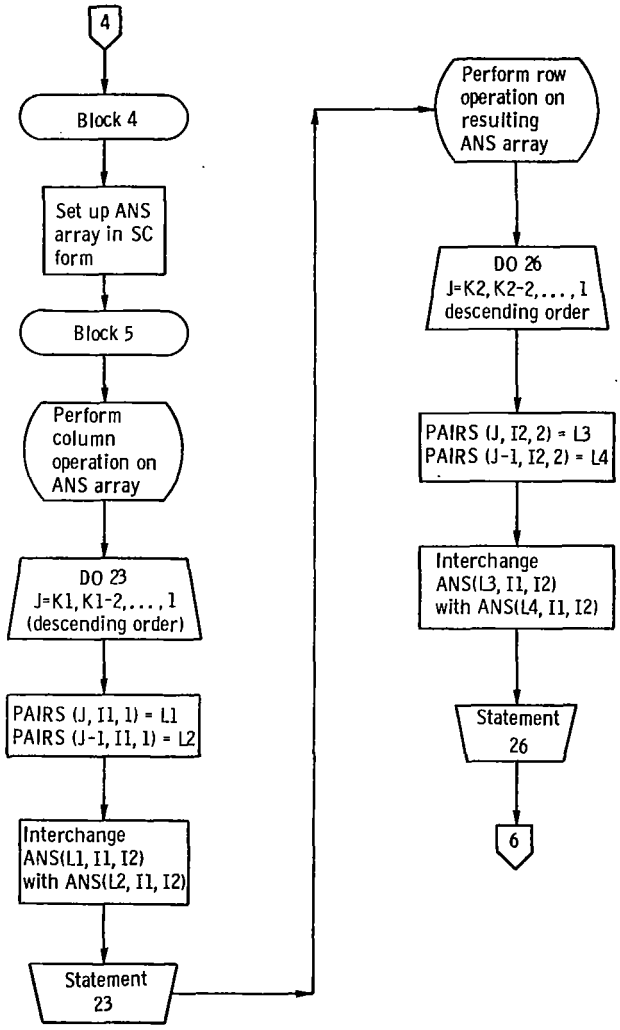
Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 2, 1969,
124-09-12-01-22.

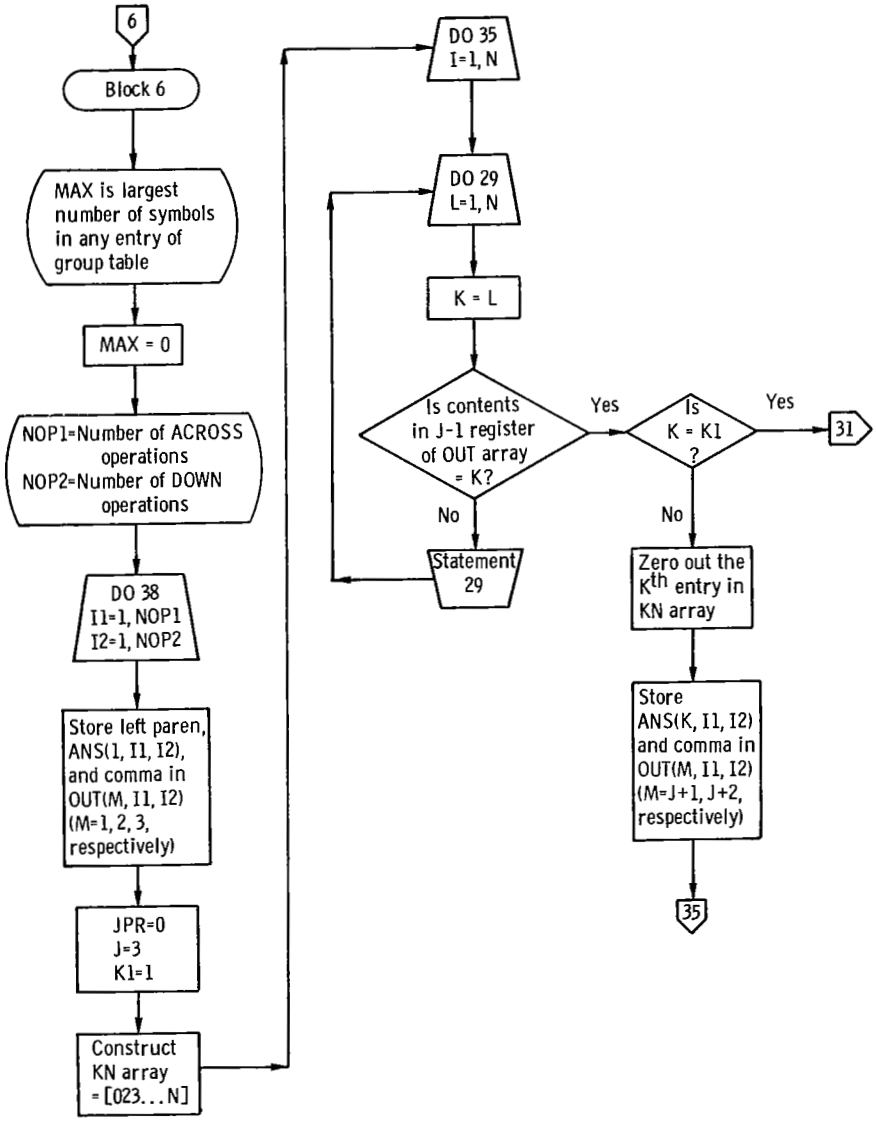
APPENDIX A

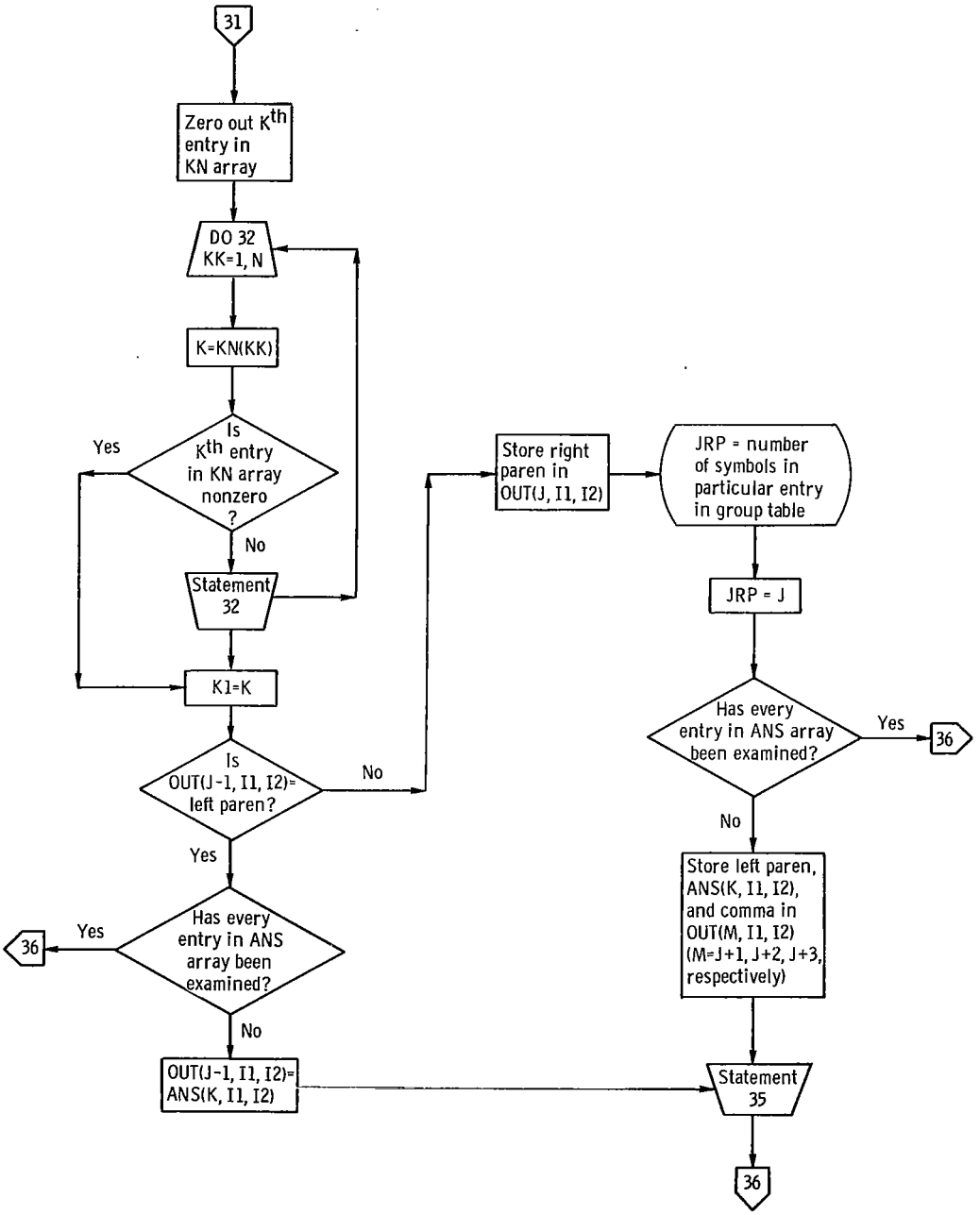
FLOW OF PROGRAM LOGIC

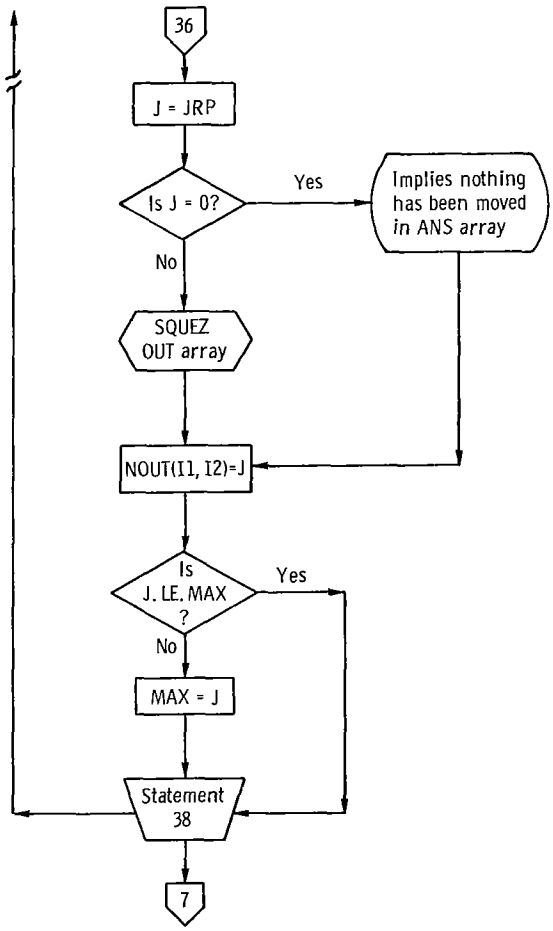


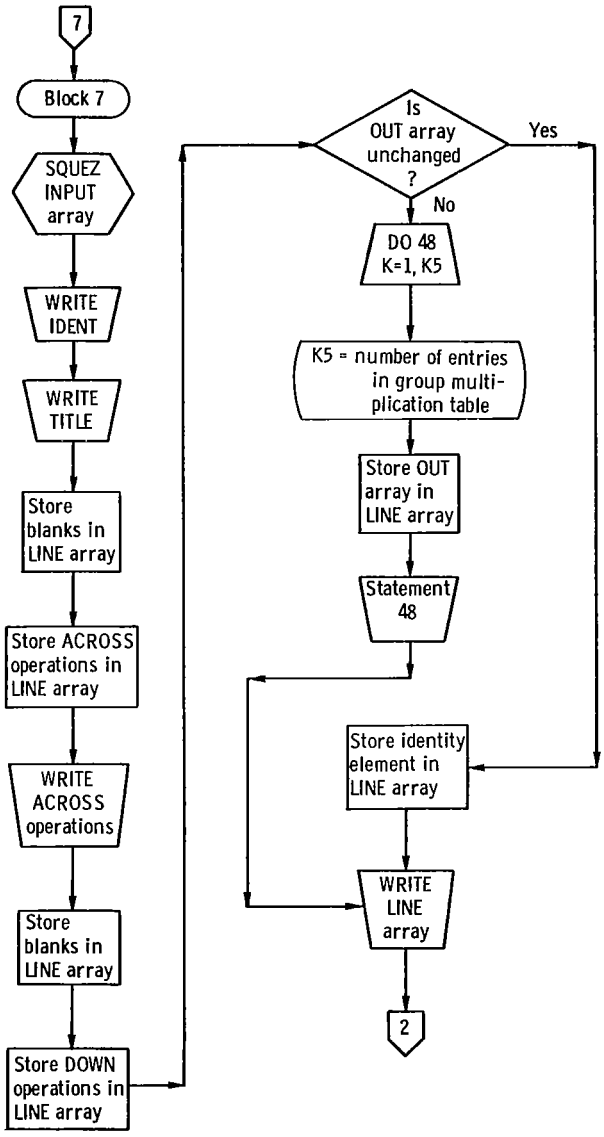

3
Block 3
DO 19
K = 1, 2
IE = NINPUT(K)
When K = 1, IE is number of symbols in ACROSS operations
When K = 2, IE is number of symbols in DOWN operations
DO 16
I = 1, IE
TEMP = INPUT(I, K)
Is TEMP = blank ?
No
Yes
16
Is TEMP = left paren ?
Yes
No
Is TEMP = right paren ?
Yes
No
Is TEMP = comma ?
Yes
No
End of operation ?
Yes
No
Construct PAIRS array
Statement 16
Count ACROSS operations; Count DOWN operations; count units
Statement 19
4
2
WRITE error messages
Yes
Check for error in ACROSS and DOWN operations
No

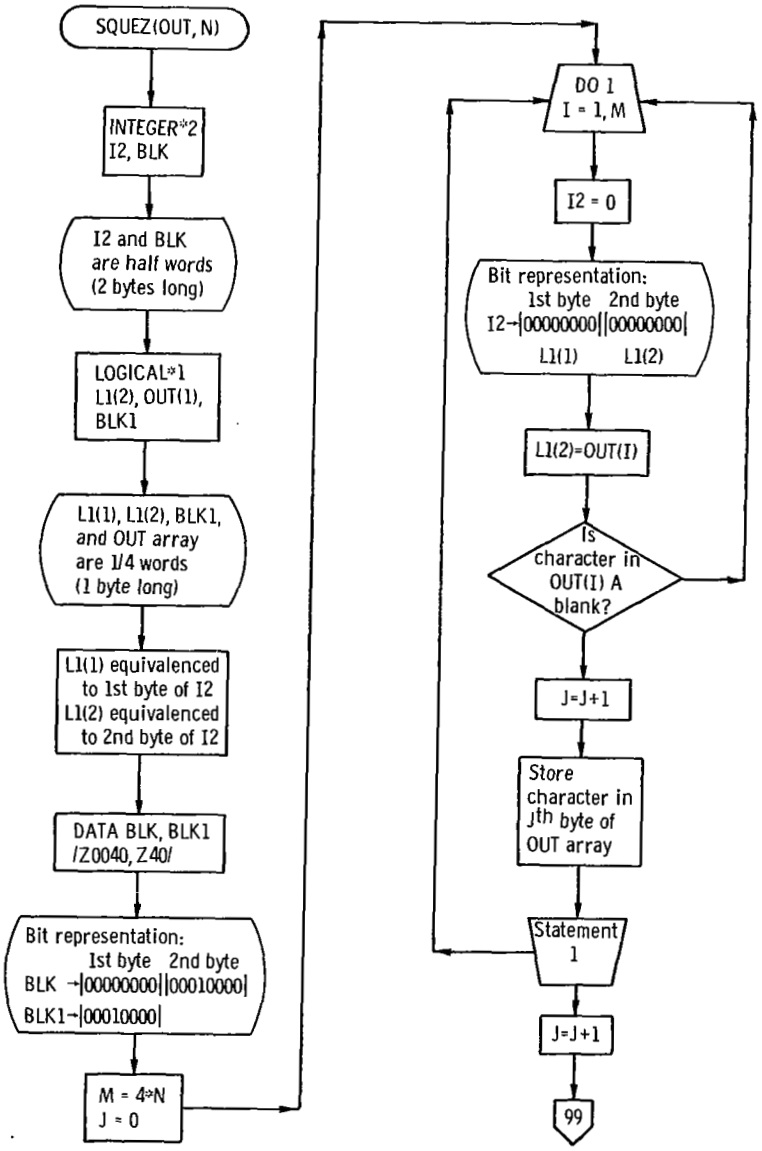

4
Block 4
Set up ANS array in SC form
Block 5
Perform column operation on ANS array
DO 23 J=K1, K1-2, . . . , 1 (descending order)
PAIRS (J, I1, 1) = L1
PAIRS (J-1, I1, 1) = L2
Interchange ANS(L1, I1, I2) with ANS(L2, I1, I2)
Statement 23
Perform row operation on resulting ANS array
DO 26 J=K2, K2-2, . . . , 1 descending order
PAIRS (J, I2, 2) = L3
PAIRS (J-1, I2, 2) = L4
Interchange ANS(L3, I1, I2) with ANS(L4, I1, I2)
Statement 26
6

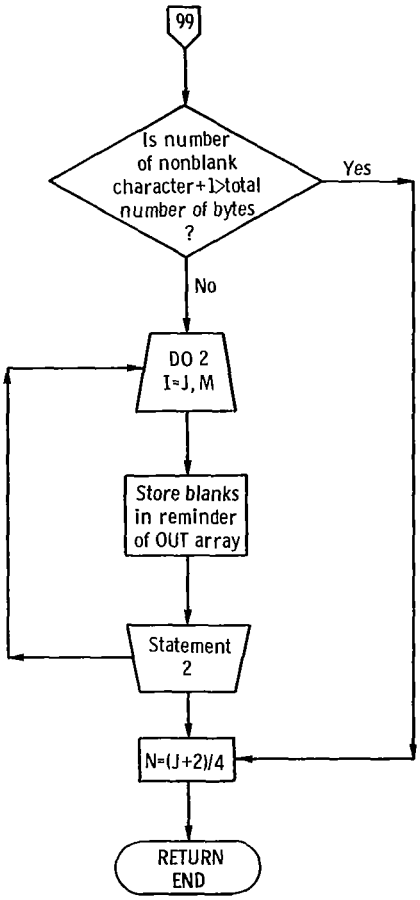

6
Block 6
MAX is largest number of symbols in any entry of group table
MAX = 0
NOP1=Number of ACROSS operations
NOP2=Number of DOWN operations
DO 38
I1=1, NOP1
I2=1, NOP2
Store left paren, ANS(1, I1, I2), and comma in OUT(M, I1, I2) (M=1, 2, 3, respectively)
JPR=0
J=3
K1=1
Construct KN array = [023...N]
DO 35
I=1, N
DO 29
L=1, N
K = L
Is contents in J-1 register of OUT array = K?
Yes
No
Statement 29
Is K = K1 ?
Yes
31
No
Zero out the Kth entry in KN array
Store ANS(K, I1, I2) and comma in OUT(M, I1, I2) (M=J+1, J+2, respectively)
35


31
Zero out Kth entry in KN array
DO 32 KK=1, N
K=KN(KK)
Is Kth entry in KN array nonzero ?
Yes
No
Statement 32
K1=K
Is OUT(J-1, I1, I2)= left paren?
No
Yes
Has every entry in ANS array been examined?
Yes
36
No
OUT(J-1, I1, I2)= ANS(K, I1, I2)
Store right paren in OUT(J, I1, I2)
JRP = number of symbols in particular entry in group table
JRP = J
Has every entry in ANS array been examined?
Yes
36
No
Store left paren, ANS(K, I1, I2), and comma in OUT(M, I1, I2) (M=J+1, J+2, J+3, respectively)
Statement 35
36


36
J = JRP
Is J = 0?
Yes
Implies nothing has been moved in ANS array
No
SQUEZ OUT array
NOUT(I1, I2)=J
Is J. LE. MAX ?
Yes
No
MAX = J
Statement 38
7


7
Block 7
SQUEZ INPUT array
WRITE IDENT
WRITE TITLE
Store blanks in LINE array
Store ACROSS operations in LINE array
WRITE ACROSS operations
Store blanks in LINE array
Store DOWN operations in LINE array
Is OUT array unchanged ?
Yes
No
DO 48 K=1, K5
K5 = number of entries in group multiplication table
Store OUT array in LINE array
Statement 48
Store identity element in LINE array
WRITE LINE array
2


SQUEZ(OUT, N)
INTEGER*2 I2, BLK
I2 and BLK are half words (2 bytes long)
LOGICAL*1 L1(2), OUT(1), BLK1
L1(1), L1(2), BLK1, and OUT array are 1/4 words (1 byte long)
L1(1) equivalenced to 1st byte of I2
L1(2) equivalenced to 2nd byte of I2
DATA BLK, BLK1 /Z0040, Z40/
Bit representation:
1st byte 2nd byte
BLK →|00000000||00010000|
BLK1→|00010000|
M = 4*N
J = 0
DO 1 I = 1, M
I2 = 0
Bit representation:
1st byte 2nd byte
I2→|00000000||00000000|
L1(1) L1(2)
L1(2)=OUT(I)
Is character in OUT(I) A blank?
J=J+1
Store character in Jth byte of OUT array
Statement 1
J=J+1
99


99
Is number of nonblank character+1>total number of bytes ?
Yes
No
DO 2 I=J, M
Store blanks in reminder of OUT array
Statement 2
N=(J+2)/4
RETURN
END

APPENDIX B

LISTING

```
C       BLOCK (1)  SETS UP CONSTANTS
C
        DIMENSION INPUT(440,2), INITAL(6), IDENT(80), NPAIR(24,2), NCUT(24
       1,24), LINE(32), NIN(24,2), NINPUT(2), NOPA(2), KN(6)
        INTEGER RP,CM,TEMP,BLK,CHAR,COL,PAIRS(18,24,2),ANS(6,24,24),CUT(15
       1,24,24),TITLE(80)
        LOGICAL COMMA,ONE
        DATA LP,RP,CM,BLK,COL,IDNT/'(',')',',',' ',':','C'/
C
C       BLOCK (2)  READS IN DATA
C
1       READ (5,66) TITLE
        READ (5,66) IDENT
        ILP=0
        IRP=0
        N=0
        CHAR=BLK
        DO 5 I=1,80
        TEMP=IDENT(I)
        IF (TEMP.EQ.BLK) GO TO 5
        IF (TEMP.EQ.LP) GO TO 2
        IF (TEMP.EQ.RP) GO TO 3
        IF (TEMP.EQ.CM) GO TO 4
        CHAR=ISLL(8,CHAR)+ISRL(24,TEMP)
        GO TO 5
2       ILP=ILP+1
        GO TO 5
3       IRP=IRP+1
4       N=N+1
        IF (N.GT.6) GO TO 61
        IF (CHAR.EQ.BLK) GO TO 62
        INITAL(N)=CHAR
        CHAR=BLK
5       CONTINUE
        IF (ILP.NE.1.OR.IRP.NE.1) GO TO 63
        ID=80
        CALL SQUEZ (IDENT,ID)
        DO 8 K=1,2
        DO 7 I=1,5
        JS=(I-1)*80+1
        JE=I*80
        READ (5,66) (INPUT(J,K),J=JS,JE)
        DO 6 J=JS,JE
        IF (INPUT(J,K).NE.BLK) GO TO 7
6       CONTINUE
        NINPUT(K)=JS
        GO TO 8
7       CONTINUE
8       CONTINUE
```

```
C
C     BLOCK (3)  STORES EACH OPERATION IN PAIRS ARRAY AS A PRODUCT OF
C                TRANSPOSITIONS
C
      DO 19 K=1,2
      NPAIR(1,K)=0
      NOP=1
      IPRN=0
      CHAR=BLK
      II=0
      ONE=.TRUE.
      COMMA=.FALSE.
      IE=NINPUT(K)
      DO 16 I=1,IE
      TEMP=INPUT(I,K)
      IF (TEMP.EQ.BLK) GO TO 16
      IF (TEMP.EQ.LP) GO TO 9
      IF (TEMP.EQ.RP) GO TO 10
      IF (TEMP.EQ.CM) GO TO 11
      IF (TEMP.EQ.COL) GO TO 14
      CHAR=ISLL(8,CHAR)+ISRL(24,TEMP)
      GO TO 15
9     IPRN=IPRN+1
      ONE=.TRUE.
      IF (IPRN.LT.0.OR.IPRN.GT.1) GO TO 57
      GO TO 15
10    IPRN=IPRN-1
      IF (.NOT.COMMA) GO TO 58
      COMMA=.FALSE.
      ONE=.TRUE.
      IF (IPRN.LT.0.OR.IPRN.GT.1) GO TO 57
      GO TO 12
11    COMMA=.TRUE.
12    NPAIR(NOP,K)=NPAIR(NOP,K)+1
      IF (NPAIR(NOP,K).GT.17) GO TO 59
      IF (CHAR.EQ.BLK) GO TO 56
      IJ=NPAIR(NOP,K)
      PAIRS(IJ,NOP,K)=CHAR
      IF (ONE) GO TO 13
      NPAIR(NOP,K)=NPAIR(NOP,K)+1
      IJ=NPAIR(NOP,K)
      PAIRS(IJ,NOP,K)=CHAR
13    CHAR=BLK
      ONE=.FALSE.
      GO TO 15
14    NOP=NOP+1
      IF (NOP.GT.24) GO TO 60
      NPAIR(NOP,K)=0
15    II=II+1
      INPUT(II,K)=TEMP
16    CONTINUE
      NINPUT(K)=II+1
      IF (INPUT(II,K).NE.COL) GO TO 17
      NOP=NOP-1
      GO TO 18
17    INPUT(II+1,K)=COL
18    IF (NPAIR(1,K).EQ.0) GO TO 1
      NOPA(K)=NOP
19    CONTINUE
```

```
C
C      BLOCK (4)  SETS UP ANS ARRAY IN STANDARD FORM OR CONFIGURATION
C
       NOP1=NOPA(1)
       NOP2=NOPA(2)
       DO 20 I1=1,NOP1
       DO 20 I2=1,NOP2
       DO 20 I=1,N
20     ANS(I,I1,I2)=INITAL(I)
C
C      BLOCK (5)  PERFORMS THE PRODUCT OPERATIONS
C
       DO 27 I1=1,NOP1
       K1=NPAIR(I1,1)
       DO 27 I2=1,NOP2
       K2=NPAIR(I2,2)
       DO 23 I=1,K1,2
       J=K1-I+1
       J1=0
       J2=0
       DO 22 L=1,N
       IF (ANS(L,I1,I2).NE.PAIRS(J,I1,1)) GO TO 21
       J1=L
21     IF (ANS(L,I1,I2).NE.PAIRS(J-1,I1,1)) GO TO 22
       J2=L
22     CONTINUE
       IF (J1.EQ.0.OR.J2.EQ.0) GO TO 55
       ITMP=ANS(J1,I1,I2)
       ANS(J1,I1,I2)=ANS(J2,I1,I2)
       ANS(J2,I1,I2)=ITMP
23     CONTINUE
       DO 26 I=1,K2,2
       J=K2-I+1
       J1=0
       J2=0
       DO 25 L=1,N
       IF (ANS(L,I1,I2).NE.PAIRS(J,I2,2)) GO TO 24
       J1=L
24     IF (ANS(L,I1,I2).NE.PAIRS(J-1,I2,2)) GO TO 25
       J2=L
25     CONTINUE
       IF (J1.EQ.0.OR.J2.EQ.0) GO TO 55
       ITMP=ANS(J1,I1,I2)
       ANS(J1,I1,I2)=ANS(J2,I1,I2)
       ANS(J2,I1,I2)=ITMP
26     CONTINUE
27     CONTINUE
```

```
C
C      BLOCK (6)  IDENTIFIES THE SINGLE GROUP ELEMENT WHICH HAS THE SAME
C                 EFFECT AS BLOCK (5)
C
       MAX=0
       DO 38 I1=1,NOP1
       DO 38 I2=1,NOP2
       OUT(1,I1,I2)=LP
       OUT(2,I1,I2)=ANS(1,I1,I2)
       OUT(3,I1,I2)=CM
       JRP=0
       J=3
       K1=1
       DO 28 KK=1,N
28     KN(KK)=KK
       KN(1)=0
       DO 35 I=1,N
       DO 29 L=1,N
       K=L
       IF (OUT(J-1,I1,I2).EQ.INITAL(K)) GO TO 30
29     CONTINUE
30     IF (K.EQ.K1) GO TO 31
       KN(K)=0
       J=J+1
       OUT(J,I1,I2)=ANS(K,I1,I2)
       J=J+1
       OUT(J,I1,I2)=CM
       GO TO 35
31     KN(K)=0
       DO 32 KK=1,N
       K=KN(KK)
       IF (K.NE.0) GO TO 33
32     CONTINUE
33     K1=K
       IF (OUT(J-2,I1,I2).NE.LP) GO TO 34
       IF (K.EQ.0) GO TO 36
       OUT(J-1,I1,I2)=ANS(K,I1,I2)
       GO TO 35
34     OUT(J,I1,I2)=RP
       JRP=J
       IF (K.EQ.0) GO TO 36
       J=J+1
       OUT(J,I1,I2)=LP
       J=J+1
       OUT(J,I1,I2)=ANS(K,I1,I2)
       J=J+1
       OUT(J,I1,I2)=CM
35     CONTINUE
36     J=JRP
       IF (J.EQ.0) GO TO 37
       CALL SQUEZ (OUT(1,I1,I2),J)
37     NOUT(I1,I2)=J
       IF (J.LE.MAX) GO TO 38
       MAX=J
38     CONTINUE
```

```
C       BLOCK (7)  SETS UP THE OUTPUT FORMAT AND WRITES ERROR MESSAGES
C
        DO 41 K=1,2
        J=0
        J1=1
        IE=NINPUT(K)
        DO 40 I=1,IE
        IF (INPUT(I,K).NE.COL) GO TO 40
        MX=I-J1+1
        CALL SQUEZ (INPUT(J1,K),MX)
        IF (MX.LE.MAX) GO TO 39
        MAX=MX
 39     J1=MX+J1
        J=J+1
        NIN(J,K)=MX
 40     CONTINUE
        IF (J.EQ.NOPA(K)) GO TO 41
 41     CONTINUE
        NC=32
        MAX=MAX+1
        NUM=NC/MAX-1
        IF (NUM.LT.2) GO TO 64
        NM=(NOP1+NUM-1)/NUM
        WRITE (6,67) (IDENT(I),I=1,ID)
        WRITE (6,68) TITLE
        K1=1
        DO 54 I=1,NM
        DO 42 L=1,32
 42     LINE(L)=BLK
        L1=MAX
        DO 44 J=1,NUM
        I1=(I-1)*NUM+J
        IF (I1.GT.NOP1) GO TO 45
        K2=NIN(I1,1)+K1-1
        DO 43 K=K1,K2
        L1=L1+1
 43     LINE(L1)=INPUT(K,1)
        L1=(J+1)*MAX
 44     K1=K2+1
 45     WRITE (6,69) LINE
        K3=1
        DO 53 I2=1,NOP2
        L1=0
        DO 46 L=1,32
 46     LINE(L)=BLK
        K4=NIN(I2,2)+K3-1
        DO 47 K=K3,K4
        L1=L1+1
 47     LINE(L1)=INPUT(K,2)
        K3=K4+1
        L1=MAX
        DO 51 J=1,NUM
        I1=(I-1)*NUM+J
        IF (I1.GT.NOP1) GO TO 52
        K5=NOUT(I1,I2)
        IF (K5.EQ.0) GO TO 49
        DO 48 K=1,K5
        L1=L1+1
 48     LINE(L1)=OUT(K,I1,I2)
        GO TO 50
```

```
   49     L1=L1+1
          LINE(L1)=IDNT
   50     L1=(J+1)*MAX
   51     CONTINUE
   52     WRITE (6,70) LINE
   53     CONTINUE
   54     CONTINUE
          GO TO 1
   55     WRITE (6,71) (IDENT(I),I=1,ID)
          GO TO 1
   56     WRITE (6,72)
          GO TO 1
   57     WRITE (6,73)
          GO TO 1
   58     WRITE (6,74)
          GO TO 1
   59     WRITE (6,75)
          GO TO 1
   60     WRITE (6,76)
          GO TO 1
   61     WRITE (6,77)
          GO TO 65
   62     WRITE (6,72)
          GO TO 65
   63     WRITE (6,73)
          GO TO 65
   64     WRITE (6,78)
          GO TO 1
   65     RETURN
   66     FORMAT (80A1)
   67     FORMAT ('1',10X,'GROUP ',11A4)
   68     FORMAT (' ',80A1)
   69     FORMAT ('-',32A4)
   70     FORMAT (' ',32A4)
   71     FORMAT (' ILLEGAL ELEMENT IN GROUP. IDENTITY GROUP IS ',11A4)
   72     FORMAT (' BLANK IS NOT A VALID ELEMENT.')
   73     FORMAT (' ILLEGAL USE OF PARENS.')
   74     FORMAT (' ILLEGAL GROUP.')
   75     FORMAT (' THE PAIRS ARRAY HAS BEEN EXCEEDED.')
   76     FORMAT (' TOO MANY OPERATIONS.  LIMIT IS 24.')
   77     FORMAT (' MORE THAN 6 ELEMENTS IN IDENTITY GROUP.')
   78     FORMAT (' NOT ENOUGH ROOM ON PRINT LINE TO PRINT TABLE.')
          END
```

```
      SUBROUTINE SQUEZ(OUT,N)
      INTEGER*2 I2,BLK
      LOGICAL*1 L1(2),OUT(1),BLK1
      EQUIVALENCE (L1(1),I2)
      DATA BLK,BLK1/Z0040,Z40/
      M= 4*N
      J=0
      DO 1 I=1,M
      I2= 0
      L1(2)= OUT(I)
      IF(I2.EQ.BLK) GO TO 1
      J= J+1
      OUT(J)= OUT(I)
    1 CONTINUE
      J = J+1
      IF(J.GT.M) GO TO 3
      DO 2 I=J,M
    2 OUT(I)= BLK1
    3 N= (J+2)/4
      RETURN
      END
```

REFERENCES

1. Norton, Lewis M.: ADEPT - A Heuristic Program for Proving Theorems of Group Theory. PhD. Thesis. Rep. MAC-TR-33, Massachusetts Inst. Tech., Sept. 1966. (Available from DDC as AD-645660.)

2. Elspas, Bernard; Goldberg, Jack; Jackson, Charles L.; Kautz, William H.; and Stone, Harold S.: Properties of Cellular Arrays for Logic and Storage. Sci. Rep. 3, Stanford Research Inst. (AFCRL-67-0463, DDC No. AD-658832), July 1967.

3. Maurer, Ward D.: Computer Experiments in Finite Algebra. Comm. ACM, ACM, 9 598 (1968), vol. 9, no. 8, Aug. 1966, pp. 598-603, 643.

4. Cannon, John J.: Computers in Group Theory: A Survey. Comm. ACM, vol. 12, no. 1, Jan. 1969, pp. 3-12.

5. Lomont, John S.: Applications of Finite Groups. Academic Press, 1959, p. 260.

6. Allen, Gabriel: An Efficient Method for Computation of Character Tables of Finite Groups. NASA TN D-4763, 1968.